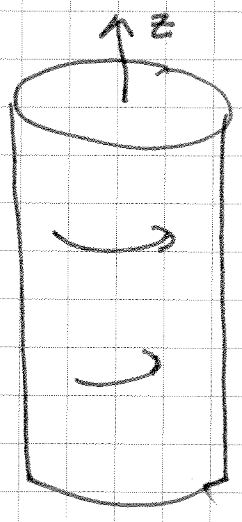
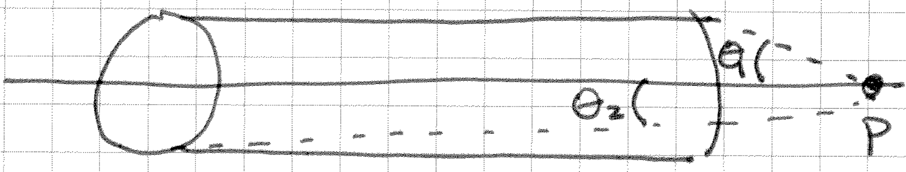
Magnetic Circuits

We have already investigated the field of an infinitely long permanent magnet with fixed magnetization density $\vec{M} = M_0 \hat{z}$



$$\vec{B}_i = \mu_0 M_0 \hat{z} \qquad \vec{K}_b = M_0 \hat{\phi}$$

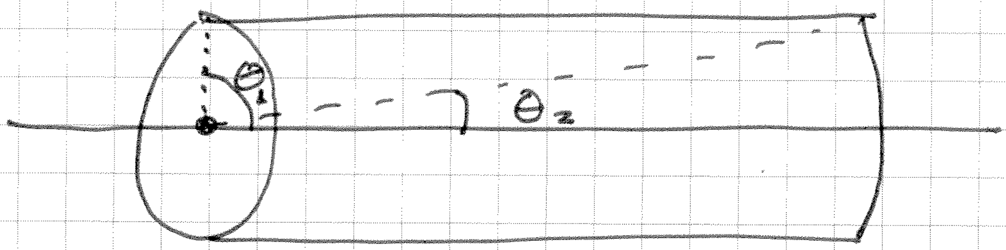
For a finite cylinder, we can use the field of a finite solenoid.



$$B_{axis} = \frac{\mu_0 K_b}{2}\left(\cos\theta_2 - \cos\theta_1\right)$$

Which can be used to examine the surface field of cylindrical magnet.

At the surface of the cylinder, if the magnet is long but not infinite



$\theta_1 = \pi/2 \qquad \theta_2 \rightarrow 0$

$$B_{end} = \frac{\mu_0 K_b}{2}\left(\cos 0 - \cos \pi/2\right)$$

$$= \frac{\mu_0 K}{2}$$

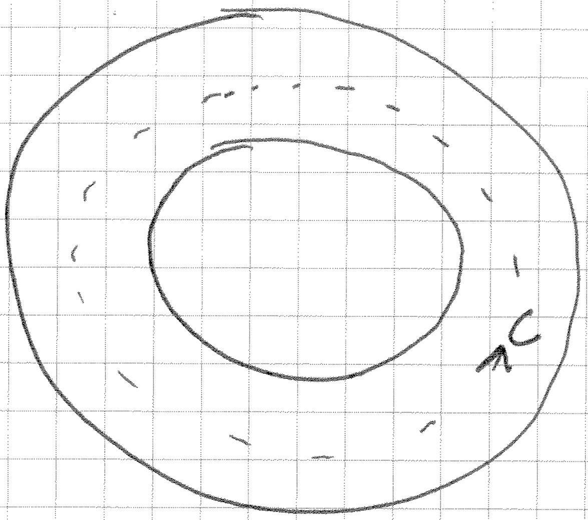
$= \frac{1}{2}$ infinite cylinder field

⟹ This is only the field along the axis; the field becomes complicated as you move off axis.

⟹ To avoid these issue, wrap the magnet in a circle.

Find $\vec{B}_i$

$$\oint_C H\cdot d\vec{l} = I_{f,enc} = 0 \qquad \Rightarrow H_i = 0$$



$$H_i = 0 = \frac{B_i}{\mu_0} - M_i$$

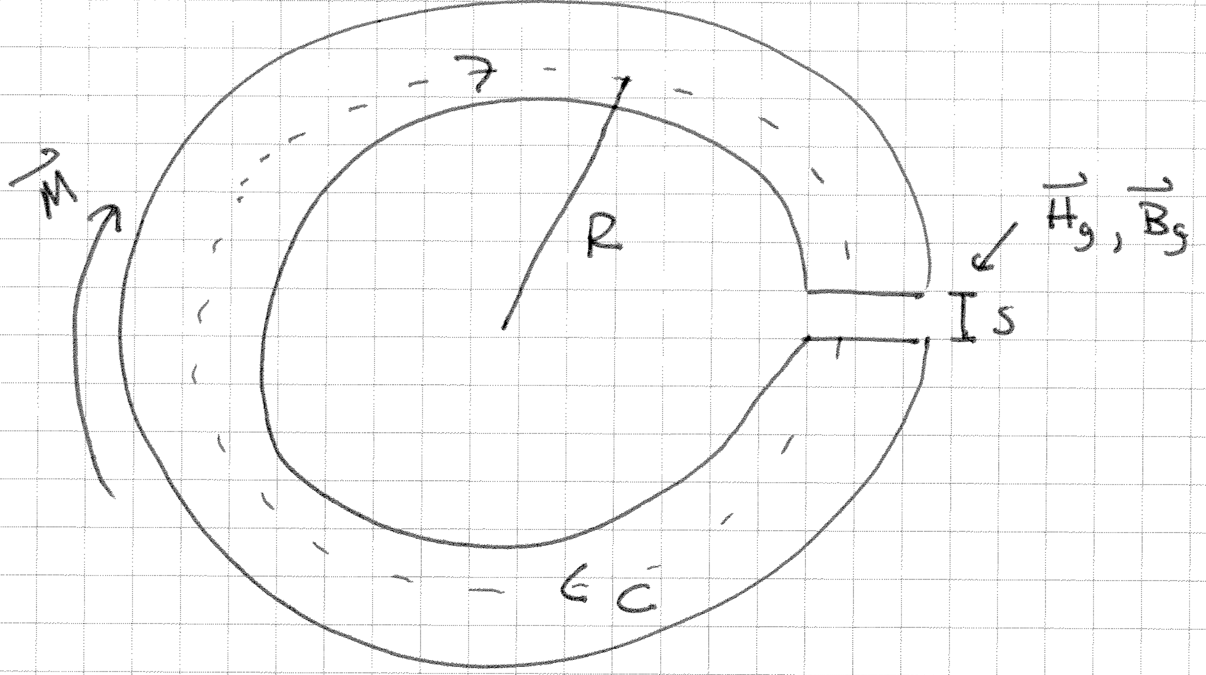
$$B_i = \mu_0 M_i = \mu_0 M$$

$\Rightarrow$ The same field as the infinite solenoid

Dfn Magnetomotive Force (mmf) (Not a force)

$$mmf = \oint \vec{H}\cdot d\vec{l}$$

Ex Permanent magnet with slit of length s cut into it.



$\vec{M}$ assumed clockwise

From infinite solenoid example, $\vec{B}$, $\vec{M}$ in same direction in magnetized material.

No free currents so

$$\text{mmf} = \oint \vec{H} \cdot d\vec{l} = 0$$

$\Rightarrow$ At slits surface $\vec{B}$ must be continuous

$$\vec{B}_i = \vec{B}_g$$

In slit, $\vec{M}_g = 0$, so

$$\vec{H}_g = \frac{\vec{B}_g}{\mu_0} = \frac{\vec{B}_i}{\mu_0}$$

Inside Magnet

$$\vec{H}_i = \frac{\vec{B}_i}{\mu_0} - \vec{M}$$

$$\text{mmf} = 0 = \oint \vec{H} \cdot d\vec{l} = (2\pi R - s) H_i + s H_g = 0$$

$\Rightarrow$ Outside magnet $\vec{B}_g \parallel \vec{H}_g$ which means $H_i < 0$ so $\vec{H}_i$ points in the opposite direction to $\vec{B}_i$.

Substitute

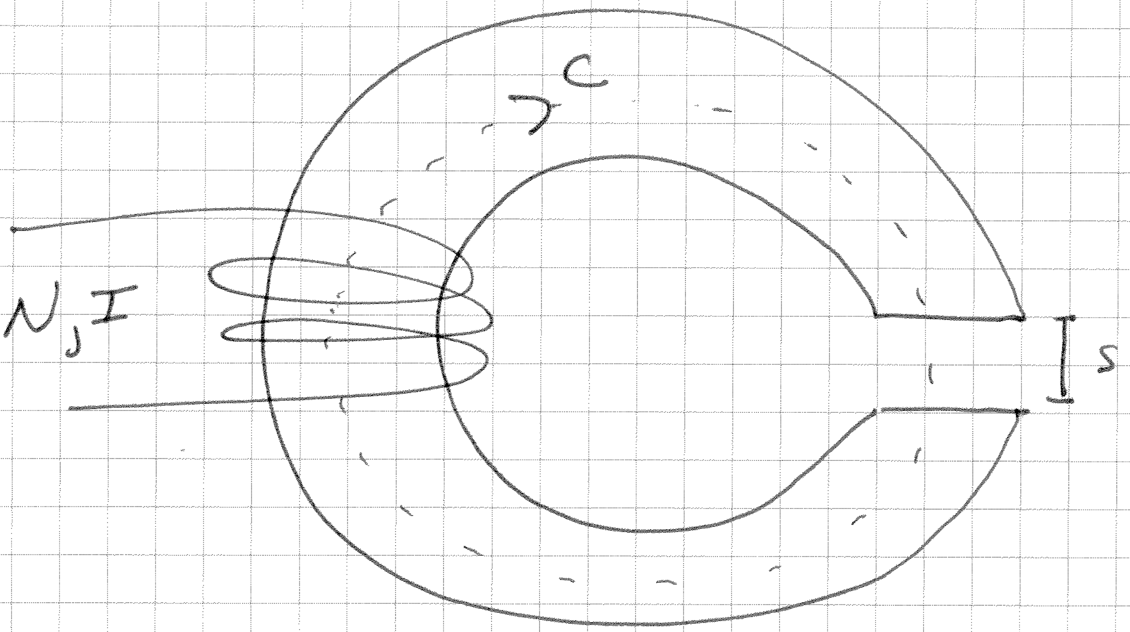
$$(2\pi R - s)\left(\frac{B_i}{\mu_0} - M\right) + s\frac{B_i}{\mu_0} = 0$$

$$2\pi R \frac{B_i}{\mu_0} = (2\pi R - s) M$$

$$B_i = \mu_0 \left(1 - \frac{s}{2\pi R}\right) M$$

⟹ The magnetic field is reduced by a factor of the ratio of the gap size to the total path length.

Ex Electromagnet, N turns, radius R, gap s with relative permeability at operating field μ_r.



As before, $B_i = B_g$

$$\text{mmf} = \oint \vec{H} \cdot d\vec{l} = I_f = NI$$

$$= H_i(2\pi R - s) + s H_g$$

$$H_g = \frac{B_g}{\mu_0} = \frac{B_i}{\mu_0}$$

$$H_i = \frac{B_i}{\mu_0 \mu_r}$$

<u>Substitute</u>

$$\frac{B_i}{\mu_0 \mu_r}(2\pi R - s) + \frac{s B_i}{\mu_0} = NI$$

$$\frac{2\pi R B_i}{\mu_r} - \frac{s B_i}{\mu_r} + s B_i = \mu_0 N I$$

$$B_i(2\pi R - s + \mu_r s) = \mu_r \mu_0 N I$$

$$B_i(2\pi R + (1 - \mu_r) s) = \mu_r \mu_0 N I$$

$$B_i = \frac{1}{1 + \frac{s(\mu_r - 1)}{2\pi R}} \cdot \underset{\substack{\uparrow \\ \text{No gap field}}}{\frac{\mu_r N I \mu_0}{2\pi R}}$$

Since μ_r can be large, this can be a large reduction.

From CRC Handbook

MAGNETIC PROPERTIES OF TRANSFORMER STEELS

Ordinary Transformer Steel

B (Gauss)	H (Oersted)	Permeability = B/H
2,000	0.60	3,340
4,000	0.87	4,600
6,000	1.10	5,450
8,000	1.48	5,400
10,000	2.28	4,380
12,000	3.85	3,120
14,000	10.9	1,280
16,000	43.0	372
18,000	149	121

High Silicon Transformer Steel

B	H	Permeability
2,000	0.50	4,000
4,000	0.70	5,720
6,000	0.90	6,670
8,000	1.28	6,250
10,000	1.99	5,020
12,000	3.60	3,340
14,000	9.80	1,430
16,000	47.4	338
18,000	165	109

SATURATION CONSTANTS FOR MAGNETI SUBSTANCES

Substance	Field intensity (For saturation)	Induced magnetization (For saturation)	Substance	Field intensity (For satur	I
Cobalt	9000	[illegible]	Nickel, hard	8000	
Iron, wrought	2000	[illegible]	annealed	7000	
cast	4000	[illegible]	Vicker's steel	15000	
Manganese steel	7000	200			

INITIAL PERMEABILITY OF HIGH PURITY IRON FOR VARIOUS TEMPERATURES

L. Alberts and B. J. Shelstone

Temperature °C	Permeability (gauss/oersted)
0	920
200	1040
400	1440
600	2550
700	3900
770	12580

MAGNETIC MATERIALS
High-permeability Materials

Material	Form	Approximate percent composition: Fe	Ni	Co	Mo	Other	Typical heat treatment °C	Permeability at $B = 20$ gausses	Maximum permeability	Saturation flux density B gausses	Hysteresis‡ loss W_h ergs/cm³	Coercive‡ force H_c oersteds	Resistivity microhm-cm	Density, g/cm³
Cold rolled steel	Sheet	98.5	—	—	—	—	950 Anneal	180	2,000	21,000	—	1.8	10	7.88
Iron	Sheet	99.91	—	—	—	—	950 Anneal	200	5,000	21,500	5,000	1.0	10	7.88
Purified iron	Sheet	99.95	—	—	—	—	1480 H_2 + 880	5,000	180,000	21,500	300	.05	10	7.88
4% Silicon-iron	Sheet	96	—	—	—	4 Si	800 Anneal	500	7,000	19,700	3,500	.5	60	7.65
Grain oriented*	Sheet	97	—	—	—	3 Si	800 Anneal	1,500	30,000	20,000	—	.15	47	7.67
45 Permalloy	Sheet	54.7	45	—	—	.3 Mn	1050 Anneal	2,500	25,000	16,000	1,200	.3	45	8.17
45 Permalloy†	Sheet	54.7	45	—	—	.3 Mn	1200 H_2 Anneal	4,000	50,000	16,000	—	.07	45	8.17
Hipernik	Sheet	50	50	—	—	—	1200 H_2 Anneal	4,500	70,000	16,000	220	.05	50	8.25
Monimax	Sheet	—	—	—	—	—	1125 H_2 Anneal	2,000	35,000	15,000	—	.1	80	8.27
Sinimax	Sheet	—	—	—	—	—	1125 H_2 Anneal	3,000	35,000	11,000	—	—	60	—
78 Permalloy	Sheet	21.2	78.5	—	—	.3 Mn	1050 + 600 Q§	8,000	100,000	10,700	200	.05	16	8.60
4-79 Permalloy	Sheet	16.7	79	—	4	.3 Mn	1100 + Q	20,000	100,000	8,700	200	.05	55	8.72
Mu metal	Sheet	18	75	—	—	2 Cr, 5 Cu	1175 H_2	20,000	100,000	6,500		.05	62	8.58
Supermalloy	Sheet	15.7	79	—	5	.3 Mn	1300 H_2 + Q	100,000	800,000	8,000	—	.002	60	8.77
Permendur	Sheet	49.7	—	50	—	.3 Mn	800 Anneal	800	5,000	24,500	12,000	2.0	7	8.3
2V Permendur	Sheet	49	—	49	—	2 V	800 Anneal	800	4,500	24,000	6,000	2.0	26	8.2
Hiperco	Sheet	54	—	34	—	Cr	850 Anneal	650	10,000	24,200	—	1.0	25	8.0
2-81 Permalloy	Insulated powder	17	81	—	2	—	650 Anneal	125	130	8,000		<1.0	10^6	7.8
Carbonyl iron	Insulated powder	99.9	—	—	—	—	—	55	132	—		—	—	7.36
Ferroxcube III	Sintered powder	$MnFe_2O_4 + ZnFe_2O_4$					—	1,000	1,500	2,500	—	1	10^8	5.0

* Properties in direction of rolling.
† Similar properties for Nicaloi, 4750 alloy, Carpenter 49, Armco 48.
‡ At saturation.
§ Q, quench or controlled cooling.

NETIC

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(or saturation)

000 400
000 515
000 1600

RITY
RES

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Den-
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g/cm³

7.88
7.88
7.88
7.65
7.67
8.17
8.17
8.25
8.27
—
8.60
8.72
8.58
8.77
8.3
8.2
8.0
7.8

7.86

5.0

MAGNETIC MATERIALS (Continued)

Permanent Magnet Alloys

Material	Percent composition (remainder Fe)	Heat treatment* (temperature, °C)	Magnetizing force H_{max}, oersteds	Coercive force H_c, oersteds	Residual induction B_r, gausses	Energy product BH_{max} × 10^{-6}	Method of fabrication†	Mechanical properties‡	Weight lb/in.³
Carbon steel	1 Mn, 0.9 C	Q 800	300	50	10,000	.20	HR, M, P	H, S	.280
Tungsten steel	5 W, 0.3 Mn, 0.7 C	Q 850	300	70	10,300	.32	HR, M, P	H, S	.292
Chromium steel	3.5 Cr, 0.9 C, 0.3 Mn	Q 830	300	65	9,700	.30	HR, M, P	H, S	.280
17% Cobalt steel	17 Co, 0.75 C, 2.5 Cr, 8 W	—	1,000	150	9,500	.65	HR, M, P	H, S	—
36% Cobalt steel	36 Co, 0.7 C, 4 Cr, 5 W	Q 950	1,000	240	9,500	.97	HR, M, P	H, S	.296
Remalloy or Comol	17 Mo, 12 Co	Q 1200, B 700	1,000	250	10,500	1.1	HR, M, P	H	.295
Alnico I	12 Al, 20 Ni, 5 Co	A 1200, B 700	2,000	440	7,200	1.4	C, G	H, B	.249
Alnico II	10 Al, 17 Ni, 2.5 Co, 6 Cu	A 1200, B 600	2,000	550	7,200	1.6	C, G	H, B	.256
Alnico II (sintered)	10 Al, 17 Ni, 2.5 Co, 6 Cu	A 1300	2,000	520	6,900	1.4	Si, G	H	.249
Alnico IV	12 Al, 28 Ni, 5 Co	Q 1200, B 650	3,000	700	5,500	1.3	Si, C, G	H	.253
Alnico V	8 Al, 14 Ni, 24 Co, 3 Cu	AF 1300, B 600	2,000	550	12,500	4.5	C, G	H, B	.264
Alnico VI	8 Al, 15 Ni, 24 Co, 3 Cu, 1 Ti	—	3,000	750	10,000	3.5	C, G	H, B	.268
Alnico XII	6 Al, 18 Ni, 35 Co, 8 Ti	—	3,000	950	5,800	1.5	C, G	H, B	.26
Vicalloy I	52 Co, 10 V	B 600	1,000	300	8,800	1.0	C, CR, M	D	.295
Vicalloy II (wire)	52 Co, 14 V	CW + B 600	2,000	510	10,000	3.5	C, CR, M, P	D	.292
Cunife (wire)	60 Cu, 20 Ni	CW + B 600	2,400	550	5,400	1.5	C, CR, M, P	D, M	.311
Cunico	50 Cu, 21 Ni, 29 Co	—	3,200	660	3,400	.80	C, CR, M, P	D, M	.300
Vectolite	30 Fe_2O_3, 44 Fe_3O_4, 26 Co_2O_3	—	3,000	1,000	1,600	.60	Si, G	W	.113
Silmanal	86.8 Ag, 8.8 Mn, 4.4 Al	—	20,000	6,000[a]	550	.075	C, CR, M, P	D, M	.325
Platinum-cobalt	77 Pt, 23 Co	Q 1200, B 650	15,000	3,600	5,900	6.5	C, CR, M	D	—
Hyflux	Fine powder	—	2,000	390	6,600	.97	—	—	.176

[a] Value given is intrinsic H_c.

* Q—Quenched in oil or water. A—Air cooled. B—Baked. F—Cooled in magnetic field. CW—Cold worked.

† HR—Hot rolled or forged. CR—Cold rolled or drawn. M—Machined. G—Must be ground. P—Punched. C—Cast. Si—Sintered.

‡ H—Hard. B—Brittle. S—Strong. D—Ductile. M—Malleable. W—Weak.

MAGNETIC SUSCEPTIBILITY OF THE ELEMENTS AND INORGANIC COMPOUNDS

The following table lists the magnetic susceptibilities of one gram formula weight of a number of paramagnetic and diamagnetic inorganic compounds as well as the magnetic susceptibilities of the elements.

In each instance the magnetic moment is expressed in cgs units.

A more extensive listing of the magnetic susceptibilities of inorganic compounds as well as those for organic compounds may be found in Constantes Selectionnees Diamagnetisme et Paramagnetisme Relaxation Paramagnetique, Volume 7. This table is abridged from the above publication by permission of the publishers.

Substance	Formula	Temp. °K.	Susceptibility 10^{-6} cgs
Aluminum (s)	Al	ord.	+16.5
" (l)	Al	....	+12.0
Fluoride	AlF_3	302	−13.4
Oxide	Al_2O_3	ord.	−37.0
Sulfate	$Al_2(SO_4)_3$	ord.	−93.0
"	$Al_2(SO_4)_3 \cdot 18H_2O$	ord.	−323.0
Ammonia (g)	NH_3	ord.	−18.0
" (aq)	NH_3	ord.	−17.0
Ammonium			
Acetate	$NH_4C_2H_3O_2$	ord.	−41.1
Bromide	NH_4Br	ord.	−47.0
Carbonate	$(NH_4)_2CO_3$	ord.	−42.50
Chlorate	NH_4ClO_3	ord.	−42.1
Chloride	NH_4Cl	ord.	−36.7
Fluoride	NH_4F	ord.	−23.0
Hydroxide (aq)	NH_4OH	ord.	−31.5
Iodate	NH_4IO_3	ord.	−62.3
Iodide	NH_4I	ord.	−66.0
Nitrate	NH_4NO_3	ord.	−33.6
Sulfate	$(NH_4)_2SO_4$	ord.	−67.0
Thiocyanate	NH_4SCN	ord.	−48.1
Americium (s)	Am	300	+1000.0
Antimony (s)	Sb	293	−99.0
" (l)	Sb	....	−2.5
Bromide	$SbBr_3$	ord.	−115.0
Chloride, tri	$SbCl_3$	ord.	−86.7
Chloride, penta	$SbCl_5$	ord.	−120.0
Fluoride	SbF_3	ord.	−46.0
Iodide	SbI_3	ord.	−147.0
Oxide	Sb_2O_3	ord.	−69.4
Sulfide	Sb_2S_3	ord.	−86.0
Argon (g)	A	ord.	−19.6
Arsenic (α)	As	293	−5.5
" (β)	As	293	−23.7
" (γ)	As	293	−23.0
Bromide	$AsBr_3$	ord.	−106.0
Chloride	$AsCl_3$	ord.	−79.9
Iodide	AsI_3	ord.	−142.0
Sulfide	As_2S_3	ord.	−70.0
Arsenious Acid	H_3AsO_3	ord.	−51.2
Barium	Ba	ord.	+20.6
Acetate	$Ba(C_2H_3O_2)_2 \cdot H_2O$	ord.	−100.1
Bromate	$Ba(BrO_3)_2$	ord.	−105.8
Barium (continued)			
Bromide	$BaBr_2$	ord.	−92.0
"	$BaBr_2 \cdot 2H_2O$	ord.	−119.0
Carbonate	$BaCO_3$	ord.	−58.9
Chlorate	$Ba(ClO_3)_2$	ord.	−87.5
Chloride	$BaCl_2$	ord.	−72.6
"	$BaCl_2 \cdot 2H_2O$	ord.	−100.0
Fluoride	BaF_2	ord.	−51.0
Hydroxide	$Ba(OH)_2$	ord.	−53.2
"	$Ba(OH)_2 \cdot 8H_2O$	ord.	−157.0
Iodate	$Ba(IO_3)_2$	ord.	−122.5
Iodide	BaI_2	ord.	−124.0
"	$BaI_2 \cdot 2H_2O$	ord.	−163.0
Nitrate	$Ba(NO_3)_2$	ord.	−66.5
Oxide	BaO	ord.	−29.1
"	BaO_2	ord.	−40.6
Sulfate	$BaSO_4$	ord.	−71.3
Beryllium (s)	Be	ord.	−9.0
Chloride	$BeCl_2$	ord.	−26.5
Hydroxide	$Be(OH)_2$	ord.	−23.1
Nitrate (aq)	$Be(NO_3)_2$	298	−41.0
Oxide	BeO	ord.	−11.9
Sulfate	$BeSO_4$	ord.	−37.0
Bismuth (s)	Bi	ord.	−280.1
" (l)	Bi	....	−10.5
Bromide	$BiBr_3$	ord.	−147.0
Chloride	$BiCl_3$	ord.	−26.5
Chromate	$Bi_2(CrO_4)_3$	ord.	+154.0
Fluoride	BiF_3	303	−61.0
Hydroxide	$Bi(OH)_3$	ord.	−65.8
Iodide	BiI_3	ord.	−200.5
Nitrate	$Bi(NO_3)_3$	ord.	−91.0
"	$Bi(NO_3)_3 \cdot 5H_2O$	ord.	−159.0
Oxide	BiO	ord.	−110.0
"	Bi_2O_3	ord.	−83.0
Phosphate	$BiPO_4$	ord.	−77.0
Sulfate	$Bi_2(SO_4)_3$	ord.	−199.0
Sulfide	Bi_2S_3	ord.	−123.0
Boric Acid	H_3BO_3	ord.	−34.1
Boron (s)	B	ord.	−6.7
Chloride	BCl_3	ord.	−59.9
Oxide	B_2O_3	ord.	−39.0
Bromine (l)	Br_2	....	−56.4
" (g)	Br_2		−73.5