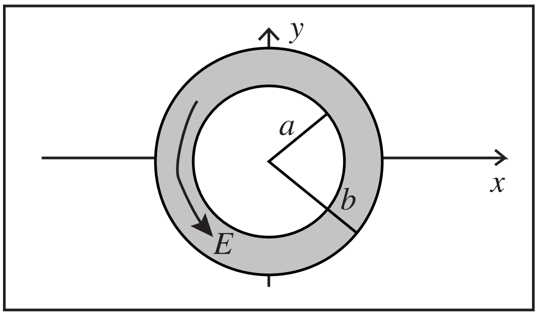
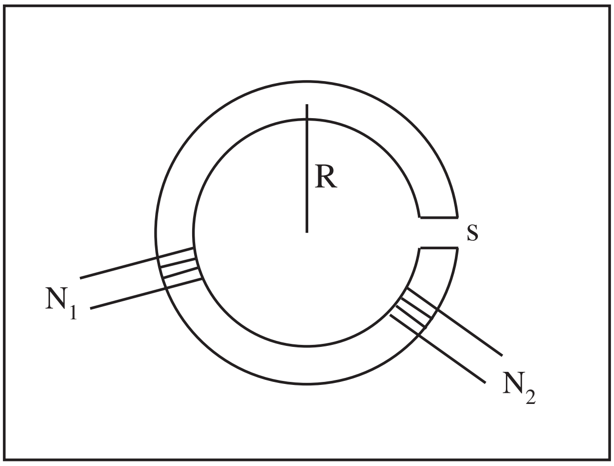
Electricity and Magnetism - Practice Final Exam - Spring 2014

Work four of the six problems. Place the problems in the order you wish them graded. The first two problems form the first half test; the second two problems form the second half test. If you turn in all six problems, then 75% of your score on the last two problems will be used to replace your lowest test score (for better of worse).

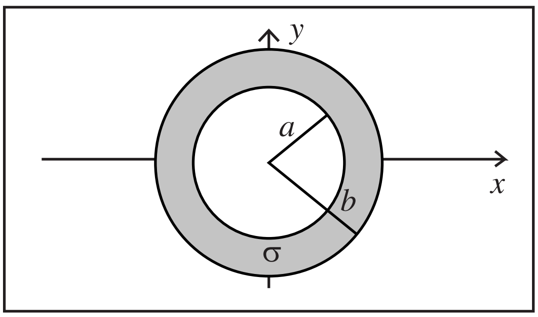
Problem 4.1 A metal washer, a thin metal disk with inner radius a, outer radius b, and thickness ℓ, is in the $x - y$ plane centered at the origin. The metal has conductivity $\sigma(s) = \gamma/s$ where γ is a constant. The region containing the disk also contains an electric field $\vec{E} = E_0\hat{\phi}$ where E_0 is a constant. Compute the magnetic field at the origin that results from the current in the disk produced by the given electric field. You may treat the current as a surface current since the disk is thin.



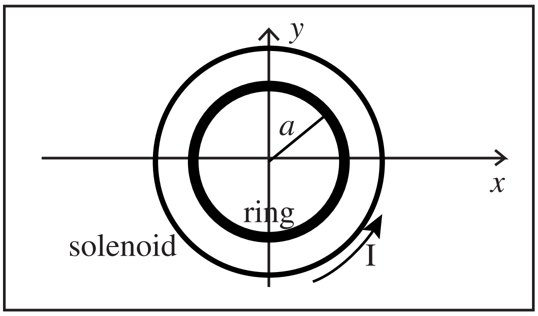
Problem 4.2 The figure below shows two coils of wire with number of turns N_1 and N_2 wound on an iron ring with relative permeability μ_r, radius R, and cross-sectional area A. The iron ring has a small gap of width s which allows the mutual inductance of the two coils to be adjusted. Compute the mutual inductance of the two coils.



Problem 4.3 A thin circular disk with inner radius a and outer radius b is in the $x - y$ plane centered at the origin. The disk has a surface charge density $\sigma(s) = \gamma s^2$ where γ is a constant. Compute the electric potential at the origin.



Problem 4.4 A long solenoid is wound with $N = 100$ turns over a distance $\ell = 20$cm. At time $t = 0$, the solenoid carries a current $I_0 = \frac{1}{4}A$. My left hand is in the solenoid adjusting, for some reason, a compass at the center of the solenoid. My wedding ring has radius of about $a = 1$cm and cross-sectional area of about $A = 1\text{mm}^2$. The ring is made of gold that has resistivity $\rho = 2.4\times 10^{-8}\Omega$m. The normal of the surface bounded by the right is parallel to the axis of the solenoid. At $t = 0$, the fuse in the meter that measures the current in the solenoid blows and the current in the solenoid decreases to zero as $I(t) = I_0 e^{-t/\tau}$ where $\tau = 1 \times 10^{-3}$s. What is the peak value of the current that is induced in my ring?



Problem 4.5 A rectangular channel of width a and height b occupies the region $0 < x < a$ and $0 < y < b$. The channel is infinite in the z direction. The $y = 0$, $y = b$, and $x = a$ sides of the channel are grounded. The $x = 0$ side has potential $V(0, y) = 0$ for $y < b/4$ and for $y > 3b/4$. In between, $V(0, y) = V_0$ for $b/4 < y < 3b/4$. Compute the potential in the channel.

Problem 4.6 A long cylindrical conductor with relative permeability μ_r and radius $s = a$ carries a total current I uniformly distributed over its cross section. The wire is co-axial with the z axis and carries current in the $+\hat{z}$ direction. Outside the wire $s > a$ the current density decays exponentially and is given by $\vec{J} = \frac{J_0}{s}e^{-s/b}\hat{z}$ where J_0 and b are constants. Compute $\vec{H}$ and $\vec{B}$ everywhere, both inside and outside the wire.