

## Electricity and Magnetism - Practice Test 1 - Spring 2014

Work four of the six problems. Place the problems in the order you wish them graded. The first two problems form the first half test; the second two problems form the second half test.

**Problem 1.1** Consider the following electromagnetic field

$$\vec{E}(x, y, z) = E_0 \sin(kx - \omega t) \hat{y}$$

$$\vec{B}(x, y, z) = B_0 \sin(kx - \omega t) \hat{z}$$

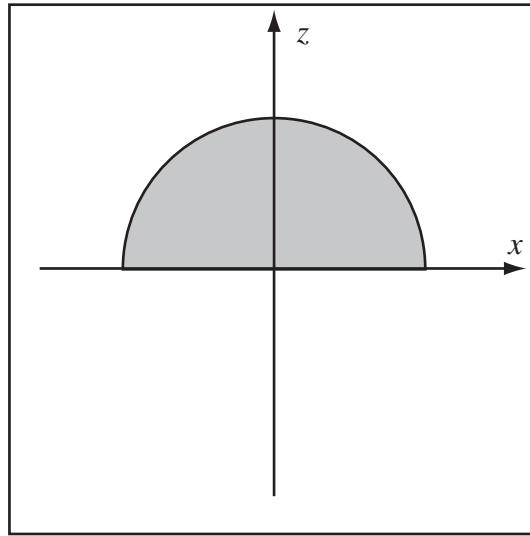
where  $E_0$ ,  $B_0$ ,  $k$ , and  $\omega$  are constants which may be related. The field exists in a region of space where both the charge density  $\rho$  and the current density  $\vec{J}$  are zero. Does this field satisfy Maxwell's equations? If it does not, state all of the equations that are not satisfied for any non-zero choice of  $E_0$ ,  $B_0$ ,  $k$ , and  $\omega$ . If the fields do satisfy Maxwell's equations, what algebraic equations must be satisfied by  $E_0$ ,  $B_0$ ,  $k$ , and  $\omega$  for Maxwell's equations to be satisfied?

**Problem 1.2** As a first model for the charge density of a semiconductor diode we could use

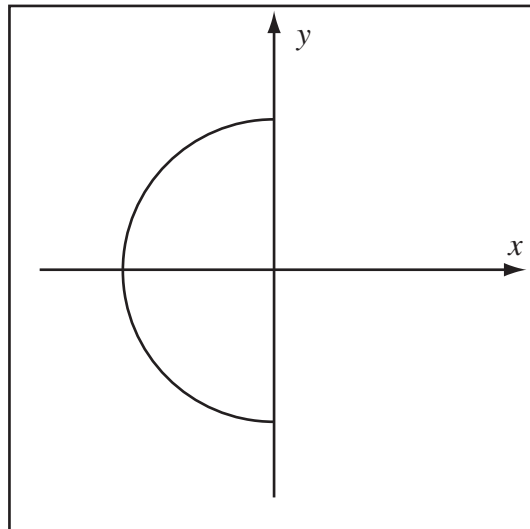
$$\rho(x) = \frac{\rho_0}{a} x \exp(-(x/a)^2).$$

Sketch this charge density and the electric field. Calculate the electric field everywhere.

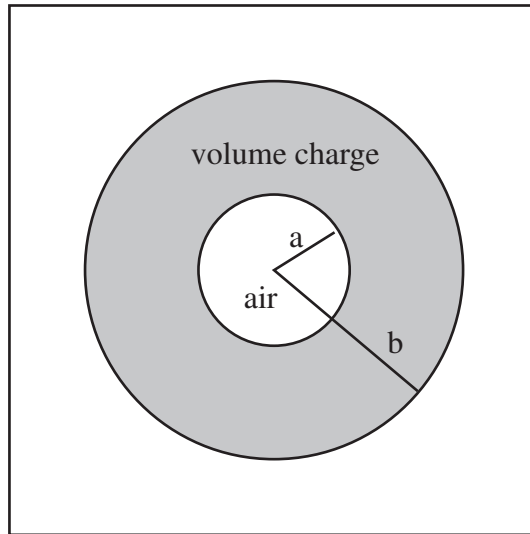
**Problem 1.3** Compute the electric field at the origin of a uniformly charged half-sphere with charge density  $\rho$  where  $\rho$  is non-zero at points  $r < a$  and  $z > 0$ .



**Problem 1.4** Compute the electric potential at a point P along the z axis,  $\vec{r}_P = (0, 0, z)$ , of a half-circle with constant linear charge density  $\lambda$ . The half-circle lies in the x - y plane, has radius a, and is non-zero for  $\pi/2 < \phi < 3\pi/2$ .



**Problem 1.5** A NON-UNIFORM spherical volume charge has charge density  $\rho(r) = \gamma r$  for  $a < r < b$  and zero otherwise;  $\gamma$  is a constant. Calculate the electric field and electric potential everywhere. Let  $V(\infty) = 0$ .



**Problem 1.6** A capacitor is formed of two concentric spherical conductors. The inner conductor has radius  $a$  and the outer conductor has inner radius  $b$  and outer radius  $c$ . Compute the capacitance between the inner and outer conductors. Compute the energy stored in the capacitor if the inner conductor has charge  $Q$  and the outer conductor charge  $-Q$  in two ways: (1) from the capacitance, (2) from the energy density of the fields.

