Quantum Mechanics

For a full definition of the electromagnetic field in terms of the potentials, we will eventually find.

$$\vec{E} = -\nabla V - \frac{\partial \vec{A}}{\partial t}$$

$$\vec{B} = \nabla \times \vec{A}$$

For the full electromagnetic field, the classical Hamiltonian for a partical of mass m moving in the field becomes

$$H = \frac{1}{2m}\left(\vec{P} - q\vec{A}\right)^2 + qV$$

= Total energy written in terms of momentum.

In quantum mechanics, the Schrodinger eqn is

$$H\psi = i\hbar\frac{\partial \psi}{\partial t}$$

Making the usual replacement, $\vec{p} \Rightarrow \frac{\hbar}{i} \nabla$ the quantum mechanical Hamiltonian for particles in electromagnetic fields is

$$H = \frac{1}{2m} \left(\frac{\hbar}{i} \nabla - q\vec{A} \right)^2 + qV$$

For example, if you wished to compute the quantized energy of an electron moving in a circular orbit in a solenoid you would use

$$V = 0 \quad , \quad \vec{A} = \gamma s \hat{\phi}$$