Test 3 Review

Magnetic Force

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

$$d\vec{F} = I\, d\vec{l} \times \vec{B}$$

$\Rightarrow$ Motion of charged particles

Current Density

$\vec{K}$, $\vec{J}$ $\qquad |\vec{J}| = \frac{I}{\text{Area}}$

$\vec{K} = \sigma\vec{v} \qquad \vec{J} = \rho\vec{v} \qquad |\vec{K}| = \frac{I}{\text{Length}}$

$\Rightarrow$ Rotational motion

$$\vec{v} = \omega r \hat{\phi}$$

Biot-Savart Law

$$\vec{B} = \frac{\mu_0}{4\pi} \int_V \frac{\vec{J} \times \hat{r}'}{r''^2}\, d\tau' \quad \text{etc.}$$

Ampere's Law

$$\nabla \times \vec{B} = \mu_0 \vec{J}$$

or

$$\oint_c \vec{B} \cdot d\vec{l} = \mu_0 I_{enc} = \mu_0 \int_s \vec{J} \cdot d\vec{a}$$

solenoids, sheets, cylinders

Vector Potential

$$\vec{B} = \nabla \times \vec{A}$$

$$\nabla \cdot \vec{A} = 0$$

$$\nabla^2 \vec{A} = -\mu_0 \vec{J}$$

$$A(\vec{r}) = \int \frac{\mu_0}{4\pi} \frac{J(\vec{r}')}{r''} d\tau'$$

$$\Phi_m = \oint_c \vec{A} \cdot d\vec{l}$$

Magnetic Moment

$$\vec{m} = NI\vec{a} = NIA\hat{n}$$

Dipole Fields

$$\vec{B}_{dip} = \frac{\mu_0}{4\pi r^3}\left[3(\vec{m}\cdot\hat{r})\hat{r} - \vec{m}\right]$$

Torque on Dipole

$$\vec{T} = \vec{N} = \vec{m} \times \vec{B}$$

Force on Dipole

$$\vec{F} = \nabla(\vec{m} \cdot \vec{B})$$

Magnetization ($\vec{M}$)

$$\vec{J}_b = \nabla \times \vec{M}$$

$$\vec{K}_b = \vec{M} \times \hat{n}$$

H - field

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

$$\nabla \times \vec{H} = \vec{J}_f$$

$$\oint_c \vec{H} \cdot d\vec{l} = I_{f enc}$$

$$\nabla \cdot \vec{B} = 0 \Rightarrow \nabla \cdot \vec{H} = -\nabla \cdot \vec{M}$$

Linear Magnetic Materials

$$\vec{M} = \chi_m \vec{H}$$

$$\vec{B} = \mu_r \mu_0 \vec{H}$$

Methods

1) Divergence / Stokes
2) Vector Identity
3) Boundary Conditions
4) $\nabla \times ? = 0 \Rightarrow$ Laplace's Eqn