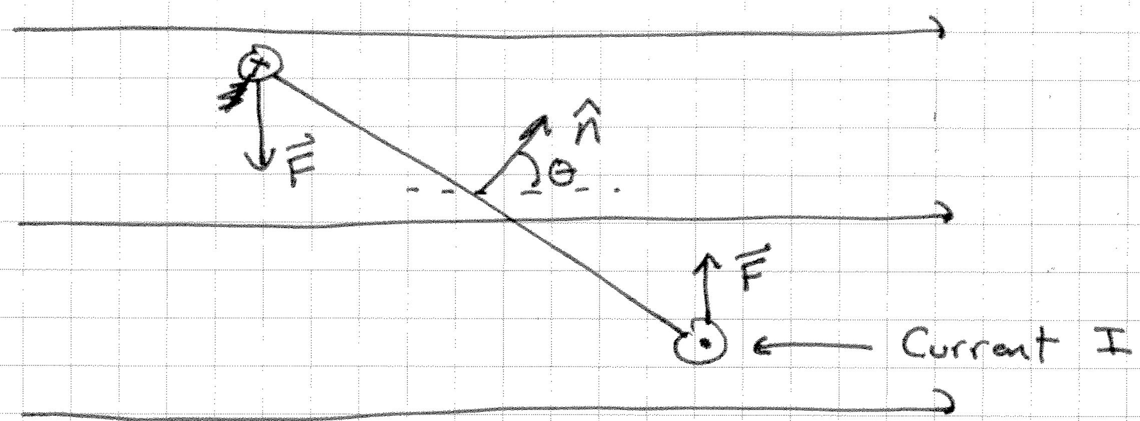
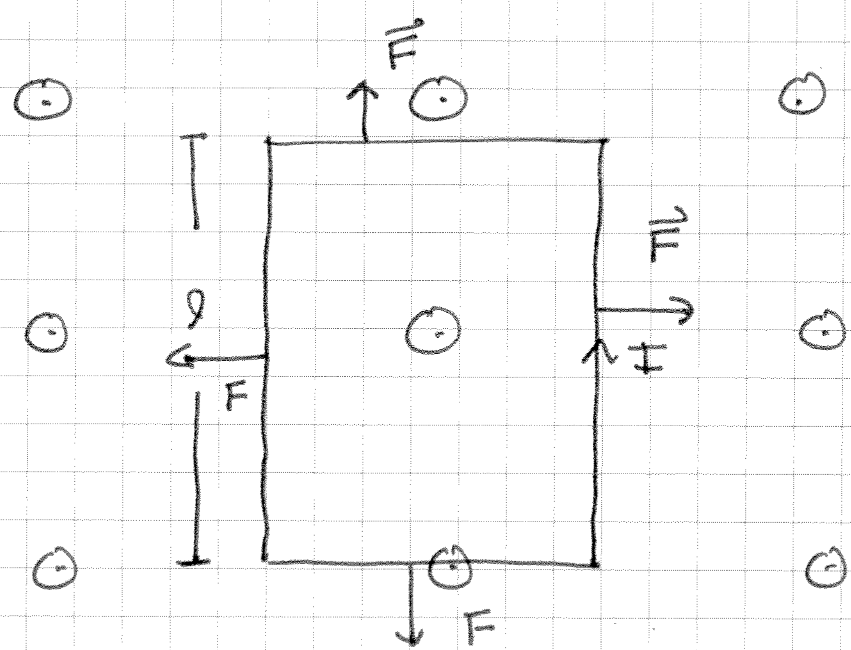
Magnetic Torque + Force on a Dipole

Consider a square $l \times l$ loop in a uniform magnetic field, $\vec{B} = B_0 \hat{x}$.

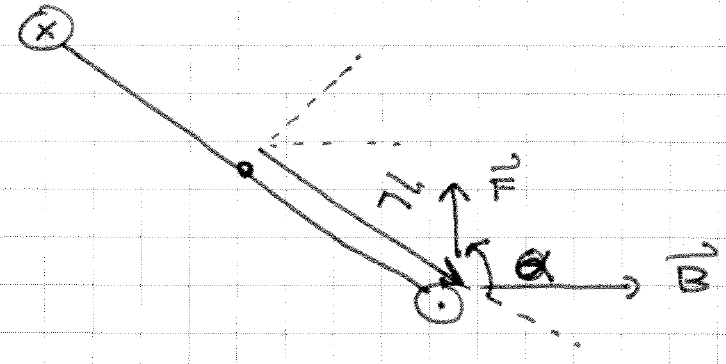


End View



$\Rightarrow$ The net force on the loop is zero, but the net torque is non-zero.

Torque ($\vec{\tau}$)



$$\vec{\tau}_1 = \vec{r} \times \vec{F}$$

$$|\vec{\tau}_1| = \frac{l}{2} F \sin\theta \quad \text{on one side}$$

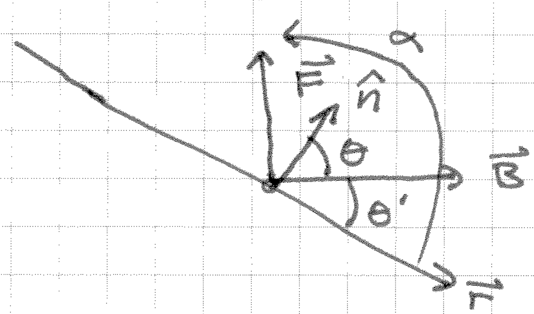
$$\text{Total Torque} = 2|\vec{\tau}_1| = l \sin\alpha F = \vec{\tau}$$

$$F = I l B$$

$$|\vec{\tau}| = \underbrace{l I l}_{m} B \sin\alpha = m B \sin\theta$$

$$\vec{\tau} = \vec{m} \times \vec{B}$$

A note on the angles



$$\alpha = \theta' + \pi/2$$

$$\theta + \theta' = \pi/2$$

$$\alpha = \pi - \theta$$

Torque on Point Dipole

$$\vec{\tau} = \vec{m} \times \vec{B}$$

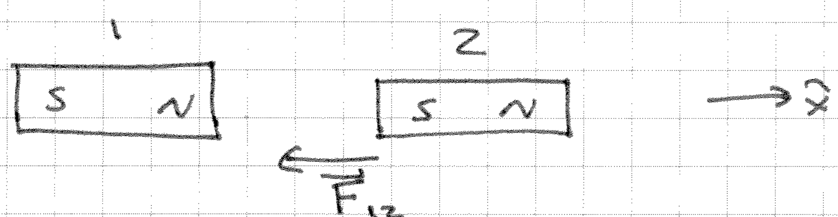
Potential Energy of Dipole (Integrate torque)

$$U = -\vec{m} \cdot \vec{B}$$

Force on Dipole

$$\vec{F} = \nabla(\vec{m} \cdot \vec{B})$$

$\Rightarrow$ This explains the odd behavior of permanent magnets.



Field of magnet 1

$$\vec{B} = \frac{2\mu_0 m_1}{r^3}\hat{x}$$

Force on Magnet 2

$$\nabla_x(\vec{m}_2 \cdot \vec{B}_1) = \nabla_x\left(\frac{2\mu_0 m_1 m_2}{r^3}\right)$$

$$= 2\mu_0 m_1 m_2 \frac{\partial}{\partial x}\frac{1}{r^3} = -4\mu_0 m_1 m_2/r^4$$

Force on Magnet 2

$$\vec{F}_{12} = \nabla\left(\vec{m}_2 \cdot \vec{B}_1\right)$$

$$= \nabla\left(\frac{2\mu_0 m_1 m_2}{r^3}\right)$$

$$= -\frac{6\mu_0 m_1 m_2}{r^4}\,\hat{x}$$

$\Rightarrow$ An r^4 dependence produces a very non-linear force.