**Heisenberg's Uncertainty Principle**

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The state of physics and chemistry prior to the twentieth century knew nothing of quantum effects, though observation and theory headed in that direction. The atom was known to contain a positively-charged nucleus surrounded by negative charge. Science did not know well the structure of either, but the nucleus defined the nature of the atom as hydrogen, copper, or so forth; the charge surrounding the nucleus was manipulated in electrical systems. Eventually scientists identified a very small particle which conveyed negative electricity—the electron. Electrons orbitted the atomic nucleus rather like planets orbit a star, but held in orbit by electric force instead of gravitational force.

Most pertinently for the purposes of this paper, this "classical" physics distinctly knew that gases were collections of particles having varying energies, and thus varying velocities. The *average* velocity of the particles of a gas corresponded to its temperature: hot gases as compared to cool gases contained more kinetic energy distributed among their particles. All particles of all matter, especially gas, exchanged energy via their frequent collisions in random motion. This regular exchange of energy implied, in Gamow's words, that "*the total energy contained in the assembly of a large number of individual particles exchanging energy among themselves through mutual collisions is shared equally (on the average) by all the particles*" (p. 7). Individual particles may deviate highly from the mean, but, statistically speaking, the system divided its total energy equally among all its particles.

In the nineteenth century, Lord Rayleigh and Sir James Jeans applied these classical principles of thermodynamics to thermal radiation. Successively hotter gases contained successively higher average energies divided among the same number of particles. That is, the total energy $E_T$ of the system, however large, was divided among the gas particles, such that the energies of the particles averaged to $E_T/n$ (for $n$ the number of particles in the gas) representing the temperature of the system. Objects emitting thermal radiation demonstrated nearly the same behavior as systems of gases: successively hotter radiating bodies emitted more radiation, representative of the greater total energy of the system, and the average frequency of the radiation increased, representative of the greater average energy of each wave. Presumably the thermal body followed the same principle as a thermodynamic system: the available energy, either heat or radiant, was distributed unequally among all available particles or frequencies.

This deduction appeared valid in all respects except for the ludicrous result it implied. In gas or some other system of matter there is a finite quantity of particles among which total energy may be divided. In radiation it is not so: there is no conceivable "upper limit" to the frequency of radiation which a body may emit. If a body's total radiant energy were distributed among all conceivable frequencies, then the average energy per frequency would be $E_T/\infty$, such that the body either must emit practically no energy at any frequency, or else must hold an infinite amount of total energy. Neither answer was satisfactory.

In 1900, Max Planck famously proposed a solution to the problem of thermal radiation, "crowning the endeavors of several centuries and involving many brilliant investigators" (Mehra, p. 241): radiation could not have any arbitrary energy associated with it. Rather, all energy existed in a finite number of "pockets" or quanta of a set amount; and the energy of a wave of radiation was proportional to its frequency but still some integer multiple of a fundamental constant. Thus the energy of a wave of radiation was $E=hf$ for the quantum constant $h$ as $6.77\times10^{-27}$ erg·s.

In light of the science of the time, as Mehra implies, Planck did not intend to spawn a new realm of physics with this work. In reality he first used the quantization of energy to derive a function relating a body's absorption and emission of radiation to its temperature and the frequency of that radiation. Prior to 1900, physicists were unsure about the relationship of "heat," "light," and other forms of radiation; Planck's work brought serious evidence that all forms of radiation were in fact the same substance, namely, propagation of energy packets through a sort of universal ether. [That radiation itself is quantized into "photons" was largely Einstein's work, not Planck's, and in any case will not be treated here.]

Into this state of physics stepped Werner Heisenberg in the 1920s. Influenced, as Gamow notes, by the recent "non-commonsensical" advances of physics such as Einstein's theory of relativity and Michelson's finite speed of light (p. 106), Heisenberg determined to think through the logical results of the new quantum physics. Classical physics held to certain ideal limiting cases which all models approached: masses could be modeled as points, trajectories could be modeled as lines, and so forth. In theory at least, the position and velocity of every particle in a system could be determined, and from this information the state of the system at any time in the future could be calculated. For Heisenberg, however, the newly discovered quantization of energy and radiation eliminated this possibility entirely.

Heisenberg considered, in the style of classical mechanics, an absolutely ideal limiting case of a particle launcher in an isolated container. (Heisenberg's "quantum microscope" is depicted in Gamow, p. 108, and is illustrated in Fig. 1 on the following page.) In Heisenberg's hypothetical design, a container is evacuated of all air and isolated from all external energy sources, so that the only force acting within the container is gravitation. A single particle is shot from some "cannon" at point C and is pulled downward by gravity. A radiating bulb capable of producing any intensity and frequency of light is placed at point B, and an ideal radiation detector is placed at point T.

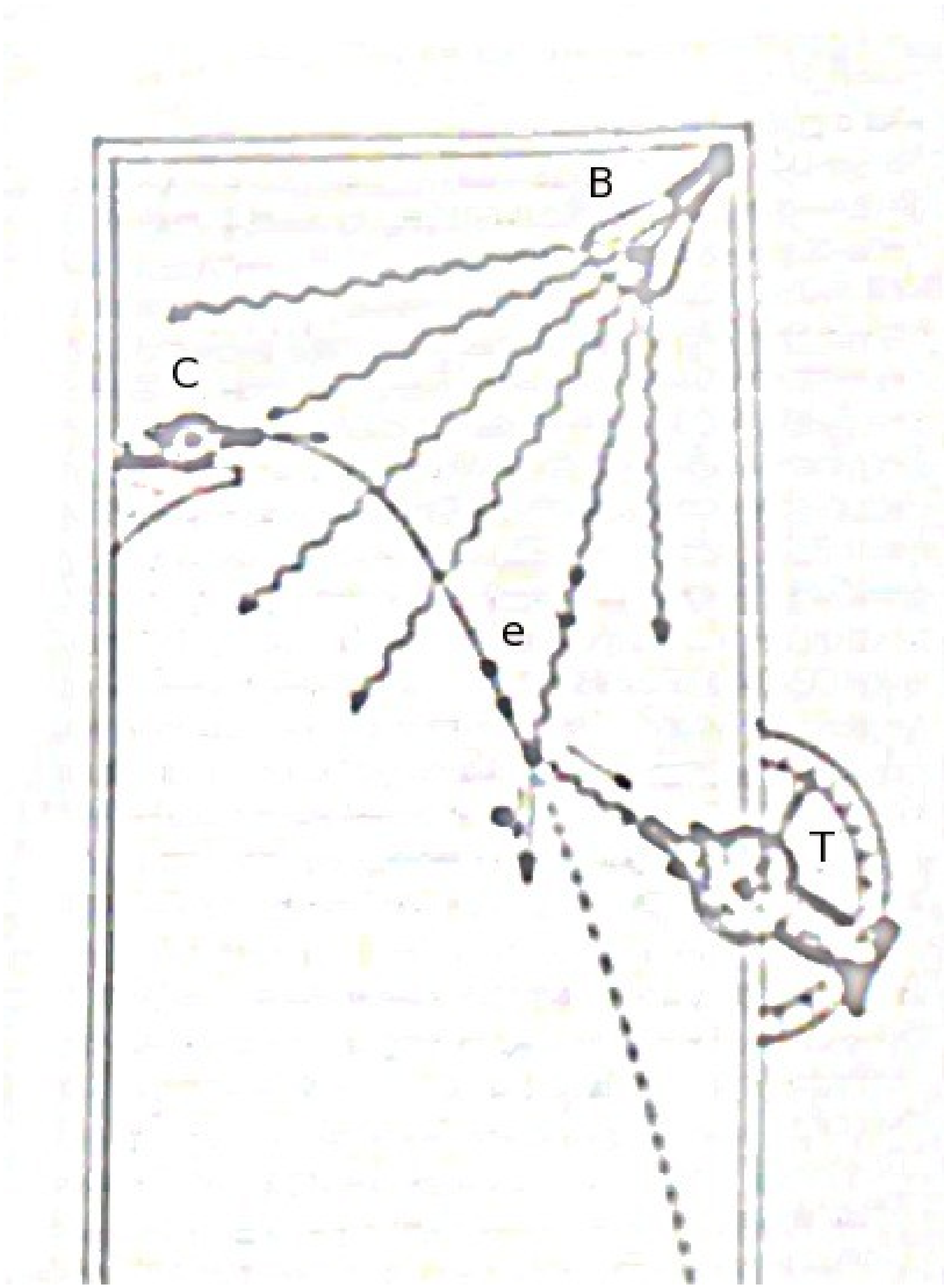


**Figure 1—Heisenberg's ideal particle detector**

**(Gamow, p. 108)**

In order to detect the particle's position at any time, the bulb is lit instantaneously to produce a single “photon” (quantum of radiant energy) of wavelength $\lambda$ . The photon is reflected by the particle into the detector, revealing the position of the particle. Two problems with this case immediately appear. In the first place, an ideal detector produces an image of size $\lambda$ for that photon; the particle, substantially smaller than $\lambda$, may be anywhere within the image. In the second place, a quantum of energy, much like the particles of a gas, will convey an unpredictable amount of its energy to a particle which it impacts; this energy changes the momentum of the particle in unpredictable ways. In classical physics the remedy to the problem is simple: the wavelength $\lambda$ can be decreased without bound to produce a more certain estimate of the particle's position, and the intensity of the light can be decreased without bound to reduce alterations to the particle's trajectory. The bulb can, in fact, be flashed as many times as desired, to produce numerous images of the particle along its trajectory.

Heisenberg noticed a problem with this ideal case. The wavelength $\lambda$ of the photon is inversely related to its frequency $f$: a smaller photon can produce a more accurate image, but this same photon oscillates at a higher frequency as well. As Planck had shown two decades earlier, the energy (or intensity) and frequency of radiation were not independent continuums. A higher frequency of photon would necessarily carry a higher energy. So then even in Heisenberg's ideal case, the bulb could not emit low-wavelength, low-intensity photons. In fact, as one reduced $\lambda$ to obtain an accurate image of the particle's position, one necessarily increased $E$ without bound. Any proportion of this ever-increasing $E$ could be conveyed to the particle: the window of possible alterations to momentum increased without bound. Likewise, as one decreased $E$ to ensure the particle held a steady velocity, one necessarily increased $\lambda$ without bound, such that the particle's position could be known less and less accurately.

Thus Heisenberg's great contribution to quantum physics was the determination that the more accurately one observed a particle's momentum, the less accurately one could observe its position, and vice versa. Mathematically, as will be demonstrated presently, the relation of the two uncertainties is simply $\Delta p \Delta q = h/4\pi$ for momentum uncertainty $\Delta p$, position uncertainty $\Delta q$, and $h$ as Planck's quantum constant (Hendry, p. 116). Furthermore, this relation is the ideal limiting case of quantum physics and cannot be improved by any system or choice of particle: "Heisenberg's uncertainty principle is a fundamental, inescapable property of the world" (Hawking, p. 57)!

Concurrently with this conceptual formulation of the uncertainty principle, Heisenberg developed a mathematical derivation for it as well. The quantum physicists were distinctly aware that classical physics stressed the continuity of all phenomena; the new physics necessitated the discreteness of all phenomena. (Albert Einstein, for instance, demonstrated that two events could not occur "simultaneously" except in a superficial way.) In classical physics one could speak of absolute position by taking a limit of velocity with respect to time, and of absolute velocity again by taking a limit of position with respect to time. In quantum physics this was impossible (Hendry, p. 113). The concept "velocity" presupposed two points and a time to pass from one to another, but in quantum physics all points and all times are discrete: limits have little absolute meaning.

Heisenberg interacted frequently with Erwin Schrödinger, Wolfgang Pauli, and Paul Dirac on the problem of defining a particle's position and velocity in quantum physics. In 1925, Heisenberg authored a paper that altered in a rudimentary fashion the calculus of classical mechanics to account for the discrete movement and radiation of an electron oscillating in its movement about a nucleus. Utilizing this new mechanics, Heisenberg, Schrödinger, and other brilliant minds discovered almost simultaneously that one highly rigorous way to enter a particle's momentum and position into the system was as infinite matrices (Ibid., p. 67ff).

Matrices of momentum and position functions served the quantized mechanics well except for a deep-seated mathematical difficulty: matrix multiplication is non-commutative. Momentum $p$ and position $q$ can be derived from one another in mechanical algorithms with no difficulty because, being scalars, $pq$ always equals $qp$ and operations return the same result regardless of which quantity is unknown. If, however, momentum and position are taken as matrices **P** and **Q**, then **PQ** differs from **QP** by some consistent quantity which must be accounted for depending on which variable is to be calculated. Mathematically, Heisenberg and Pauli postulated, the difference between the two must be the identity matrix ***I*** multiplied by the constant $h/2\pi$ (Gamow, p. 105).

Working this postulate into the mechanical formulas produced a system of equations which agreed precisely with experimental results as well as with Schrödinger's prior work in the seemingly unrelated field of wave mechanics. (Schrödinger later proved, in fact, that his functions describing the oscillations of waves were mathematically identical to Heisenberg's functions describing the oscillations of electrons.) Since the difference between **PQ** and **QP** had to be $h/2\pi$ , it followed that a corresponding relationship existed between the uncertainty of measurements of **P** and **Q**: namely, some imprecision, as a *basic property of the universe*, would exist in both matrices, on the order of $h/2\pi$ !

There were, to be sure, mixed feelings toward this idea of particle uncertainty, both in the popular mind and among scientists. The eminent Einstein, although he contributed substantially to the development of the new physics, never grew comfortable with it, and always rather resented the Heisenberg uncertainty that it implied. He famously quipped in a letter to a

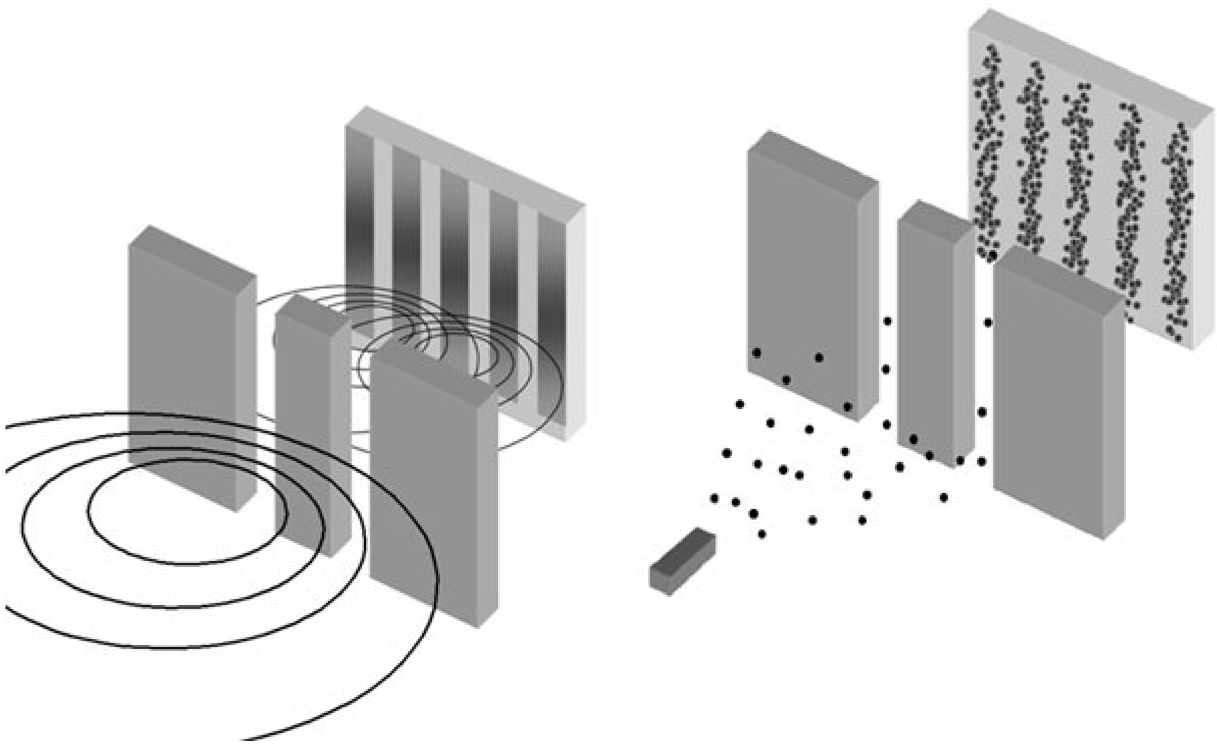

**Figure 2—Waves and electrons display similar behavior when passed through two slits in a solid barrier. Heisenberg's uncertainty principle provides a partial explanation for the behavior of electrons in this experiment. (Images from BlackLight Power, Inc.)**

colleague that "the [quantum] theory says a lot, but does not really bring us any closer to the secret of [God]. I, at any rate, am convinced that *He* does not throw dice" (Einstein to Born, 1926). The dream of naturalistic philosophy since the Scientific Revolution had been, after all, a completely determined universe. Given the state of any system in the universe at some arbitrary time, its state at any time in the future ought to be knowable, if the universe follows consistent natural laws. The uncertainty principle inherent in the new quantum physics seemed to ruin that prospect: if measuring precisely the present state of the universe were impossible, then predicting its future state would be even more so (Hawking, p. 57).

On the superatomic scale, as further study indicated, Heisenberg's uncertainty meant almost nothing. The uncertainty relation $\Delta \mathrm{p} \Delta \mathrm{q} = h/4\pi$ takes *p* as momentum, which is the product of mass and velocity. As in Gamow (p. 111) one can divide out the mass component to produce $\Delta \mathrm{v} \Delta \mathrm{q} = h/4\pi m$, indicating that the measurement uncertainty is inversely proportional to the mass of the particle, noting again that *h* is on the order of $10^{-34}\ J\cdot s$. For an electron of mass $10^{-27}$g this measurement uncertainty is larger and holds more energy than some whole atoms. For a bullet of mass 1mg, however, the measurement uncertainty is roughly three millimeters per year of velocity, and the diameter of an atom of position. Clearly, regardless of quantum uncertainty, classical mechanics remains an adequate approximation of reality on large scales.

Heisenberg's uncertainty principle caused a greater stir—one which continues to the present day—in its implications regarding the nature of matter and energy. Classical physics viewed matter as a collection of discrete particles, whether atoms or something smaller; in contrast to matter stood the nebulous entities known as waves which had no true boundary but simply propagated as disturbances through matter or ether. Yet, as has been shown, Planck from

1900 onward had quantized energy into discrete packets and identified all radiation as the same sort of wave. Einstein extended this quantization such that radiation itself represented quanta having the properties of particles. Heisenberg's uncertainty, by contrast, dismantled the idea of matter as individually observable particles. Since a particle's momentum and position could not be simultaneously known, all particles would in fact behave like waves propagating through some imprecisely-bound region of space (see Fig. 2 above)!

In light of these developments the quantum physicists could not agree among themselves as to what distinction, if any, existed between particles and waves. The eminent minds of the late 1920s held numerous conferences to discuss the correct philosophy and application of the new uncertainty principle, and particularly the wave-particle duality. Eventually Heisenberg, Dirac, Pauli, and many others ventured on a compromise of application in which, more or less, any of several different algorithms governing wave-particle behavior could be used depending on the situation under consideration. Philosophical questions pertaining to ontology, determinism, causation of events, and so forth were left to individual belief. This compromise has since become known as the "Copenhagen interpretation" of quantum physics and represents the mainstream (but not by any means the *only* stream) of scientific thought on matters of quantum dynamics (Hendry, p. 127).

Whatever its interpretation, this fundamental uncertainty principle which originated in the calculations of the unwitting Max Planck could not, after its discovery, be ignored. The new quantum physics trod new intellectual territory, territory which its pioneers were ever mapping. The chief author of the uncertainty principle, Werner Heisenberg, contributed along with other leaders (Schrödinger, Pauli, et al.) to this theory to account for phenomena where classical physics failed utterly: on the microscopic scale, where waves and particles intertwine.

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