University Physics II Honors Project

Inductance in the Power Industry

An Analysis on Transformers and Inductors

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Physic 2074

Honors Lab 1

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April 25th, 2012

1. Introduction:

Today's society relies greatly upon energy; without energy and an infrastructure to carry it, modern civilization would quickly fall. Historically, electricity has been generated through a medium such as wind, water at high speeds, combustion, thermal energy from the earth, and heavy metal radiation. All of these mediums require one important element to operate: magnets. The medium merely turns magnets, either directly or by heating water under pressure, within a coil of wire or solenoid inductor. Electric motors and generators operate on the principal of inductance. Furthermore, voltage can be increased or decreased using mutual inductance and this is also essential in energy production.

2. Purpose:

The performed project was to design, build, and implement an effective solenoid inductor out of common items for cost efficiency. Furthermore, the goal was to attain necessary supporting data that would prove that a solenoid transformer could effectively transform voltage and current through the constructed inductors.

3. Theory:

The inductor is governed by Faraday's Law, which according to Vladimir Leus of the Sobolev Institute of Mathematics and Stephen Taylor of the University of Liverpool, "The main idea of Faraday's law is that a changing magnetic flux through a surface induces an EMF in any boundary path of that surface, and a changing magnetic field induces a circulating electric field" (Leus and Taylor, 2011). The way this comes into effect in the inductor is that the magnetic field is changing according to the variable power provided by a wave generator. As the magnetic field changes, the flux changes; this leads to an induced current in the solenoid within the inductor.

According to Richard L. Coren, in his book: Basic Engineering Electromagnets, Michael Faraday established that as current moves through a coil of wire it results in a magnetic field similar to a bar magnet and the reverse will result in electricity being produced (Coren 38-39). So from an inductor energy can be converted into a magnetic field and back into current in another source; this is how the inductor and transformer operate.

The Encyclopedia Britannica defines inductance as a characteristic of a generally coiled conductor that sustains a varying current which forms a varying magnetic field that induces an electromotive force in a separate conductor within the field of the first inductor (Encyclopedia Britannica). What this means is that as a varying current is run through a coil, such as the solenoid inductor made from the toilet paper roll, it forms a varying magnet field. As this field acts on the inner solenoid, made from a pen, there is a varying magnetic flux that acts through the area of the pen; as a result it induces a current within the pen solenoid.

By definition from the Encyclopedia Britannica, the inductance from one inductor through another is called mutual inductance, this is “exemplified in a transformer” (Encyclopedia Britannica). Meaning voltage and current can be transferred from one inductor to another, or through a transformer voltage and current can be increased or decreased.

3.1 Mutual Inductance

The mutual inductance through two inductors is defined in most general physics books as the change in flux of one over the current of the other.

$$M_{AB} = M_{BA} = \frac{\phi_A}{I_B} = \frac{\phi_B}{I_A} \qquad (1)$$

Where *M* stands for mutual inductance, ϕ is the magnetic flux through the inductor, and *I* is the current. This is also the ratio of voltages between the two inductors, equation 1A shows the relationship:

$$M_{AB} = \frac{\Delta V_{Secondary}}{\Delta V_{Primary}} = \frac{\Delta V_B}{\Delta V_A} \quad (1A)$$

This comes from the definition of inductance where inductance is the flux through a surface divided by the current of the inductor. Equation 2 shows this:

$$L = \frac{\phi_m}{I} \quad (2)$$

I is the current, which for this experiment is alternating.

3.2 Alternating current

The current, I, is defined by the relationship of potential difference over resistance, equation 3:

$$I = \frac{\Delta V}{R} \quad (3)$$

For an alternating current the current varies as a sine curve. The initial current is defined as I_o and it is multiplied by the sine of the angular frequency, ω, multiplied by time, t.

$$I = I_o \sin(\omega t) = I_o \sin(2\pi f t) \quad (4)$$

Since angular frequency is defined as two pi times the frequency.

For this project only the max current is used, so equation 4 can be simplified into equation 4A:

$$I = I_o \sin(\omega t) = I_o \quad (4B)$$

Since the peak of sine is 1.

3.3 Magnetic Flux

The magnetic flux is significantly more complex. Equation 5 begins the definition of flux:

$$\phi_m = N \oint_S (\vec{B} \cdot \hat{n}) dA \qquad (5)$$

Where *N* is the number of loops in the coil being affected by the magnetic field, $\vec{B}$, $\hat{n}$ is the surface normal, and *dA* is the change in area.

The magnetic field is defined by the Biot-Savart Law using the infinite solenoid approximation it is thence equation 6:

$$\vec{B} = \left(\frac{N}{\ell}\right) \mu_0 I \qquad (6)$$

Where *N* is the number of loops of the solenoid, ℓ is the length of the solenoid, μ_o is the permeability of free space, and *I* is the current. Since this field is denoted as a vector it must have direction. The magnetic field generated by a solenoid will point completely parallel to the length of the solenoid.

When finding flux the dot product of the field and the normal must be found. The normal is a vector that points completely parallel or normal to the surface through which the field acts. The dot product is shown in equation 7:

$$(\vec{B} \cdot \hat{n}) = |\vec{B}||\hat{n}| cos\theta \qquad (7)$$

The θ is the angle between the field and the normal. In the experiment the direction of the field is always kept parallel to the normal and thus the cosine of θ is 1 and the magnitude of $\hat{n}$ is just 1. This gives equation 7A:

$$(\vec{B} \cdot \hat{n}) = |\vec{B}| \qquad (7A)$$

From this the equation for flux becomes:

$$\phi_m = N\left|\vec{B}\right| \oint_S \; dA \tag{5A}$$

The surface integral of the surface is just the area from this equation 5 becomes equation 8

$$\phi_m = N\left|\vec{B}\right| \oint_S \; dA = N_1\left|\vec{B}\right|A \tag{5B}$$

Therefore

$$\phi_m = N_s\left(\frac{N}{\ell}\right)\mu_0 IA \tag{8}$$

Finally, the flux through the surface is found to be the number of turns of wire for the surface in which the flux acts upon, multiplied by the area of that surface and then multiplied by the field of the solenoid.

3.4 Inductance

As mentioned in equation 2, inductance is defined as the magnetic flux divided by current, applying equation 8 forms equation 9.

$$L = \frac{\phi_m}{I} = \frac{N_s\left(\frac{N}{\ell}\right)\mu_0 IA}{I}$$

$$L = N\left(\frac{N}{\ell}\right)\mu_0 A \tag{9}$$

Thus the inductance is just the magnetic field acting on the number of turns of wire and the area of the surface for this specific model.

3.5 Electromotive Force

By Faraday's law for an inductor the voltage through an inductor will be the electromotive force or the time rate of change in flux. Therefore,

$$emf = -\frac{d\phi_m}{dt} = -L\frac{dI}{dt} = \Delta V \tag{10}$$

What this means is that the induced voltage or electromotive force is equal to the changing magnetic flux or the inductance multiplied by the changing current. Thus induced voltage is directly related to the time rate of change in current. The variable power source used for the experiment provides a changing current over time and since it travels through an inductor it will induce a voltage in the solenoid inductor. Since the electromotive force is with respect to a changing current and the alternating current is a sine function, the voltage can be easily found with basic calculus.

$$emf = -\left(N\left(\frac{N}{\ell}\right)\mu_0 A\right)\frac{d}{dt}\left[I_o \sin(2\pi f t)\right] \tag{11}$$

Take the derivative and get Equation 11A

$$emf = 2\pi f\left(N\left(\frac{N}{\ell}\right)\mu_0 A\right) I_o \cos(2\pi f t) \tag{11A}$$

For this project, the current and voltage where taken at the max to show maximum output potential, thus the max voltage must be found. This occurs when cosine of the angle is 1, since it is a cosine curve. Therefore, equation 12 is the max output voltage or electromotive force.

$$emf_{max} = 2\pi f\left(N\left(\frac{N}{\ell}\right)\mu_0 A\right) I_o \tag{12}$$

Finally, a proficient means to determine the effective voltage output for the solenoid constructed for this project is established.

4. Process:

To make the inductor, a toilet paper roll was used as a medium to roll high gauge speaker wire around it; forming a solenoid. The insulator in the wire was removed at the ends to allow a flow of current. Next a smaller solenoid was made using a Bic ® pen and wrapping the same type of speaker wire around it. Then an iron nail was inserted into the pen to add strength to the magnetic field through the inductor. The constructed products are shown in figure 4.1 below:

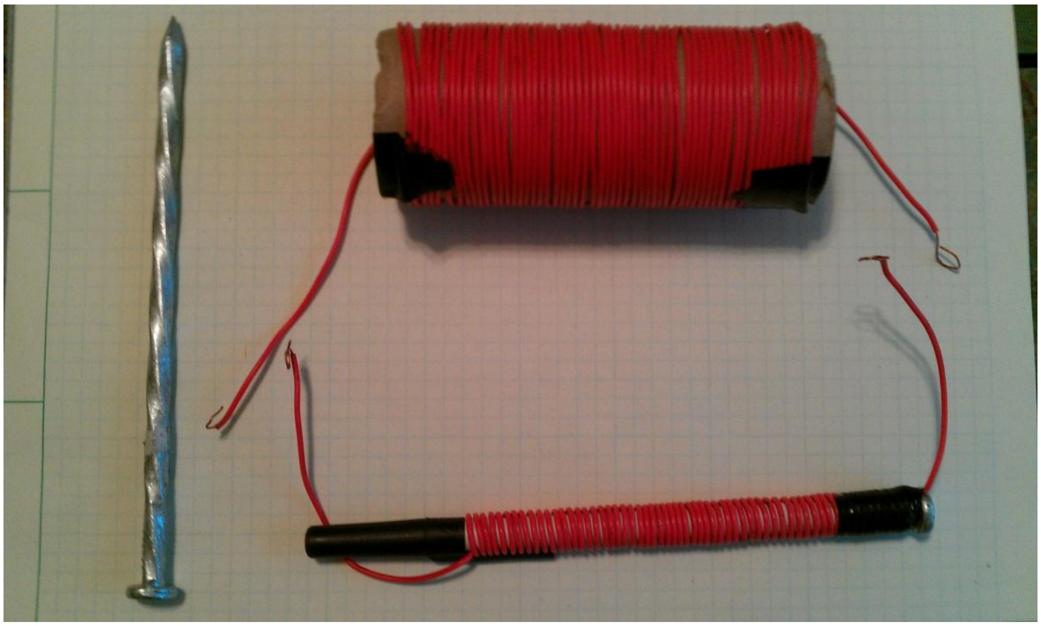


Figure 4.1: Solenoid, Inductor, and Ferromagnetic Nail

To find the effective inductance trough the inductor, the number of turns of wire around the inductor, the effective length, and the radius were all needed. Furthermore, these same measurements were needed for the solenoid. Then through calculation the inductance and mutual inductance can be found. With the inductance, the output voltage and current can be calculated. The procedure used to measure these is illustrated in the figure below:

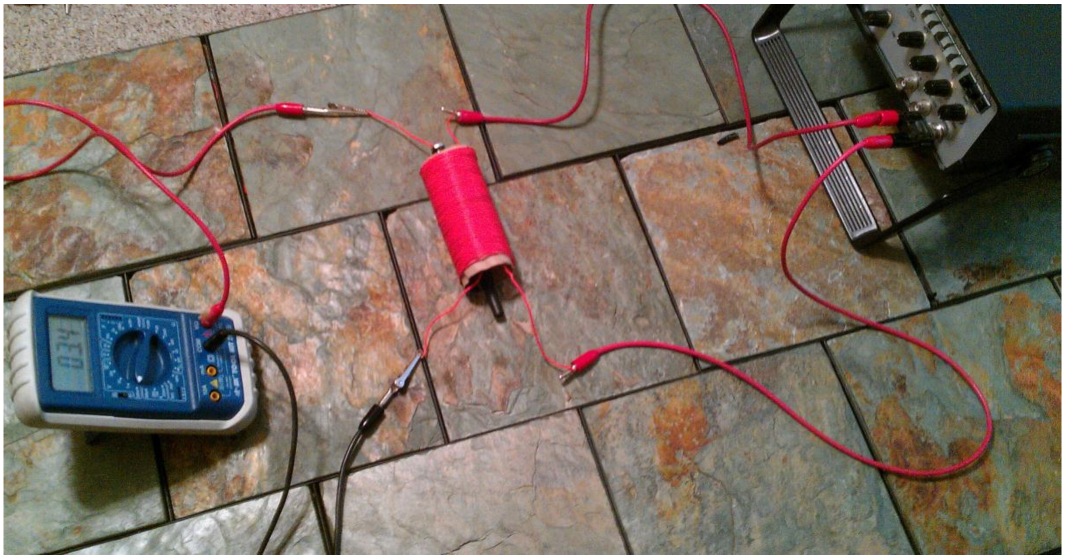


Figure 4.2: Measuring Voltage and Current

With the collected data, calculation and interpretation are all that remains.

5. Data:

For the experiment data was collected for inductance and the transformer.

The wave generator, inductor, and solenoid each output the following in the tables below as they were measured for various parameters.

Table 5.1 Specifications of Solenoid and Inductor

Specifications	**Solenoid**	**Inductor**	**Units**
# of Turns (N)	59	56	Turns
Effective Length (l)	0.1065	0.095	Meters
Radius (r)	0.0035	0.02	Meters
n (N/l)	553.991	589.474	Turns/Meters

Table 5.2 Lone Wave Generator, Inductor, and Solenoid

Measurement	Wave Generator	Inductor	Solenoid
Current (A)	0.1299	N/A	N/A
Voltage (V)	0.607	N/A	N/A
Resistance (Ohms)	2.0	1.6	0.5
Frequency (Hz)	60	N/A	N/A

Table 5.3 Minimum Amplitude at 60 Hz

Measurement	Wave Generator	Inductor	Solenoid
Current (A)	0.0107	0.0017	0.000001
Voltage (V)	0.05	0.0048	0.0002

Table 5.4 Medium Amplitude at 60 Hz

Measurement	Wave Generator	Inductor	Solenoid
Current (A)	0.0611	0.0606	0.0000016
Voltage (V)	0.287	0.0275	0.0016

Table 5.5 Maximum Amplitude at 60 Hz

Measurement	Wave Generator	Inductor	Solenoid
Current (A)	0.1299	0.1287	0.0000034
Voltage (V)	0.607	0.0563	0.0034

From this the magnetic field, inductance and flux was calculate for their respective amplitudes, the results are displayed in table 5.6:

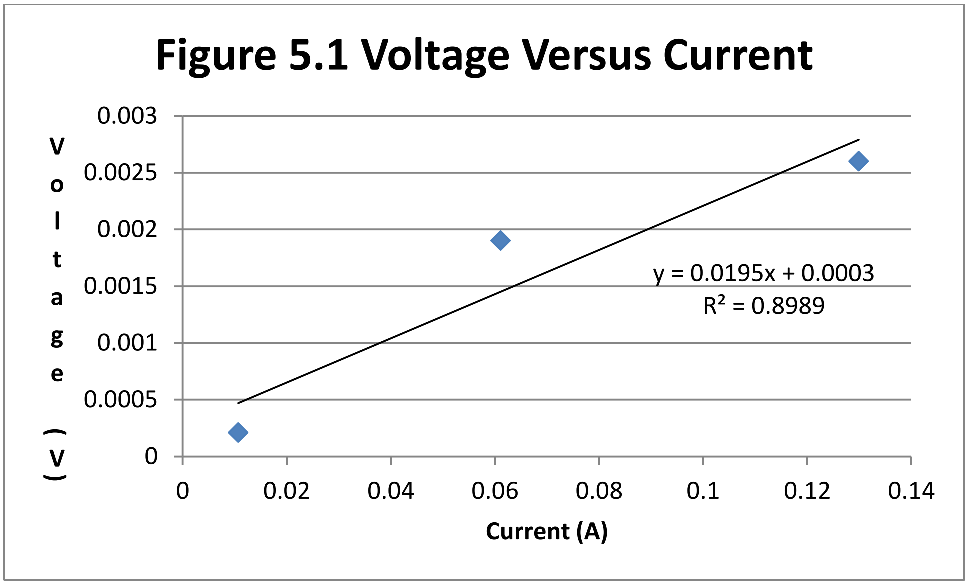
Table 5.6 Field, Flux, and Inductance

Amplitude	Current (A)	Magnetic Field (μT)	Magnetic Flux (μWb)	Inductance (μH)
Minimum	0.0107	7.93	0.558	52.15
Medium	0.0611	45.3	3.19	52.17
Maximum	0.1299	96.2	6.77	52.13

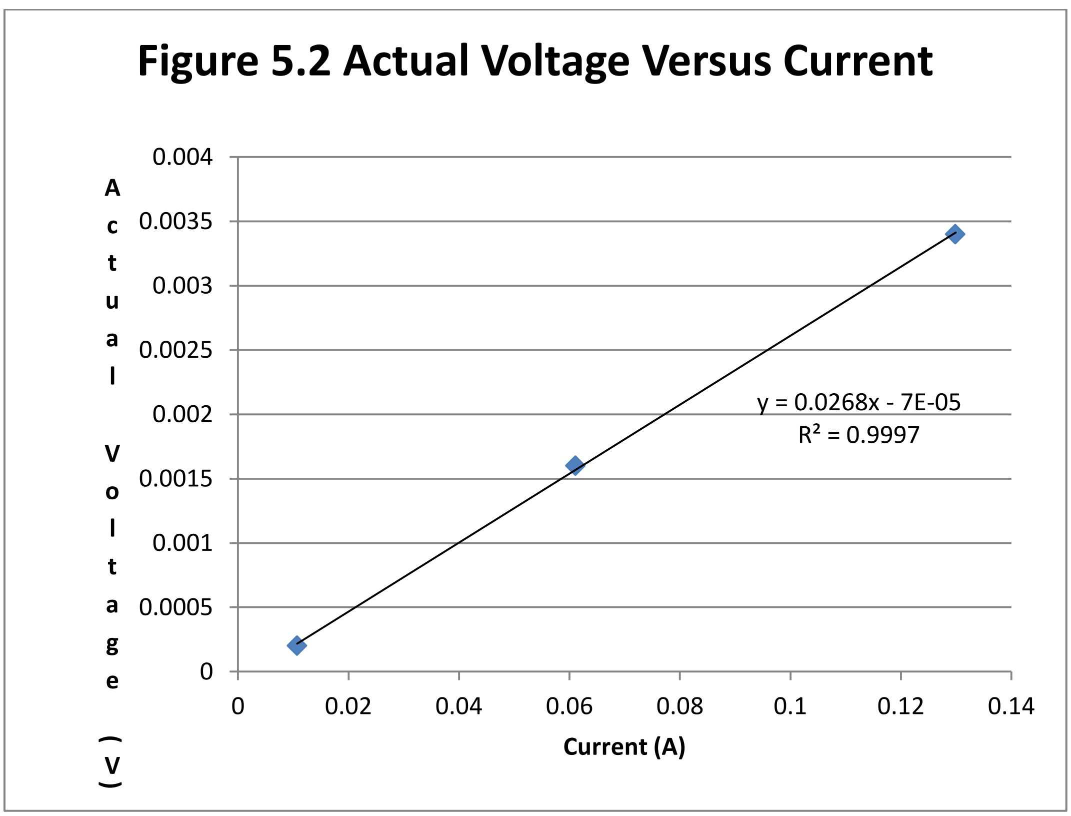
From these, the output voltage was calculated for each amplitude setting. The results are given in table 5.7 with a graphical evaluation in Figure 5.1 below.

Table 5.7 Output voltages

Amplitude	Current (A)	Output Voltage (V)	Actual Voltage (V)	%Error
Minimum	0.0107	0.00021	0.0002	5.00
Medium	0.0611	0.0019	0.0016	15.8
Maximum	0.1299	0.0026	0.0034	23.5



From the figure a linear trend can be seen of where the voltage should be as current increases. However, as the current increased so did the error. The actual voltage versus current graph is in figure 5.2 and shows what the trend should be.



This trend is a significantly better trend line and is what the ideal voltage would be.

6. Data Interpretation:

The first table just shows the initial details and will not be directly evaluated; it is just to show how efficient the experiment is. For the calculation of inductance all three amplitudes are approached identically, for simplicity only the medium output inductance will be calculated.

6.1 Inductance at 60 Hz Supplied Frequency and Medium Amplitude

With the 60 Hz frequency set and amplitude held constant in the wave generator, the inductor has a field running through it. This can be calculated using equation 5

$$\vec{B} = \left(\frac{N}{\ell}\right)\mu_0 I = \left(\frac{56}{0.095m}\right)\left(4\pi x 10^{-7}\ \frac{Tm}{A}\right)(0.0611\text{A}) = 45.3\ \mu\text{T} \quad (5)$$

Flux is governed by equation 7, when acting on the solenoid N_s is the number of turns of wire for the solenoid and A is the area of the solenoid.

$$\phi_m = N_s\left(\frac{N}{\ell}\right)\mu_0 IA = 56(45.3\ \mu\text{T})\pi(0.02m)^2 = 3.19\ \mu Wb \quad (7)$$

Inductance is governed by equation 8

$$L = N\left(\frac{N}{\ell}\right)\mu_0 A = (56)(45.3\ \mu\text{T})\pi(0.02m)^2 = 52.17\ \mu\text{H} \quad (8)$$

With these values the mutual inductance can be calculated between the solenoid and the Inductor.

6.2 Mutual Inductance Between the Inductor and the Solenoid

From equation 1 the mutual inductance or ratio of voltages can be found

$$M_{IS} = M_{SI} = \frac{\phi_I}{I_s} = \frac{\phi_s}{I_I} \quad \text{(1A)}$$

With the flux from the inductor and the current from the solenoid the mutual inductance can easily be found:

$$M = \frac{\phi_I}{I_s} = \frac{\Delta V_S}{\Delta V_I} = \frac{3.19\ \mu Tm^2}{1.6\ \mu A} = 1.994$$

Thus the mutual inductance is a ratio of 1.994 to 1.

6.3 Calculating the Output for the Mutual Inductor System.

The output voltage by Faraday's Law is the electromotive force or the negative time rate of change in flux as in equation 12:

$$emf_{max} = (2\pi)(60\ Hz)\left(\frac{(56)^2}{0.095\ m}\right)\left(4\pi x 10^{-7}\ \frac{Tm}{A}\right)(\pi(0.02\ m)^2)(0.0611\ A) = 1.9\ mV$$

This voltage is around the recorded value and is in error.

6.3 Error in the experiment

As this experiment was done on a poor college student's budget it was subject to a high capacity for error. The percent error for the medium output is shown below.

$$\%error = \frac{|0.0019 - 0.0016|}{0.0019}(100\%) = 15.8\%$$

This is a sizable error.

7. Results:

By experimentation, the current theories on inductance have been tested with results in error an average of 10.11%, a sizable error. Regardless, the effectiveness of a solenoid is shown through experimentation. The cost for the project was low since the wave generator and multi-meter were provided. In the end what is shown is that an effective inductor and transformer can be made from low cost materials and still perform on an effective scale.

8. Error analysis:

With an experimental error on average of 10.11% as calculated from the linear trend, there must be a significant source of error. The error is a result of faulty material used to build the inductors and the fact that the flux will not be completely transferred from one solenoid to the other. There are small deviations since the field is subject to outside interference. The largest source of error comes from the fact that the solenoid's magnetic field must be modeled by the infinite solenoid approximation. To correct for this error would require content beyond the scope of the University Physics II course. The corrected field would draw the output voltages closer to their actual value, resulting in smaller error.

9. Conclusion:

The performed project successfully shows that an effective transformer can be made from simple homemade inductors. There is a marginable error from the approximation of field and from errors within the equipment. Regardless, by experimentation an effective solenoid can be made and used to transform voltage or current from one inductor to another. This is how the modern power industry can send out high voltage energy and then step it down so that it can be used in the household. Transformers and inductors are an invaluable part of modern society and the more efficient a transformer can be built the cheaper energy can be provide to civilization. From small scale inductors to large industrial transformers; inductance is a pivotal part of modern society.

10. Appendix A: References

1. *Encyclopædia Britannica Online*, s. v. "inductance," accessed April 20, 2012, http://www.britannica.com/EBchecked/topic/286711/inductance.
2. Vladimir Leus and Stephen Taylor, "On the Motion of the Field of a Permanent Magnet," European Journal of Physics 32(2011) 1179-1192, accessed April 20, 2012, doi:10.1088/0143-0807/32/5/006. http://0-iopscience.iop.org.library.uark.edu/0143-0807/32/5/006/pdf/0143-0807_32_5_006.pdf
3. Coren, Richard. *Basic Engineering Electromagnetics: and Applied Approach.* New Jersey: Prentice Hall, Inc., 1989.