Sarah Withem

Dr. John Stewart

Honors University Physics II

April 15, 2009

Honors Project: The Casimir Effect

The Casimir effect is a strange attractive force that occurs when two parallel uncharged metallic planes are microscopic distances from one another in a vacuum. It also occurs in many other instances and is present in everyday life. However, calculations for the Casimir effect at larger distances are impossible to make since the understanding of macroscopic systems is extremely limited (Levin 3). The Casimir force can be pretty strong and "at separations of 10 nm - about a hundred times the typical size of an atom - the Casimir effect produces the equivalent of 1 atmosphere of pressure" (Lambrecht), which does not sound like much, but it plays a large role and contributes a large amount of friction and inoperability to nanotechnology because it causes the pieces to stick together.

In 1948 while studying van der Waals forces and colloids in the Netherlands, Hendrik Casimir's colleague Theo Underbeek noticed that the current theory used to explain van der Waals forces did not explain the results of the experiments performed on the colloids. Colloids are "viscous materials, such as paint and mayonnaise, that contain micron-sized particles in a liquid matrix" (Lambrecht) and the van der Waals force is the force that occurs between the atoms of those materials. After prodding from Underbeek, Casimir further explored the subject. Working with Dirk Polder, Hendrik Casimir discovered that the interaction between molecules could be explained if the fact that light travels at a finite speed was taken into account. This seemingly simple thought completely changed the way that the situation was viewed. Later, Casimir noticed that it all could be related to vacuum fluctuations, and further explored the possibilities of two parallel mirrors in a vacuum facing each other. He theorized that there would be an attractive force between the two mirrors. This force and its effect were named the Casmir force and Casimir effect after their discoverer. Because its conditions for observation are not easily achieved, the effect was not measured until ten years later, and as new technology was developed, there was a wave of research and measurements began in the late 1990's (Lambrecht).

Despite the recent boom in research revolving around the Casimir effect due to the continuing advancements being made in nanotechnology, the Casimir effect is a mysterious force that remains widely debated, and some even debate its very existence. But, the equations defining the Casimir effect and its force are surprisingly accurate for a wide variety of materials and shapes. From its origin in the mid 20th century, to its theory and uses in today's world, the Casimir effect is an interesting phenomenon that is sure to be the center of much attention in the coming years.

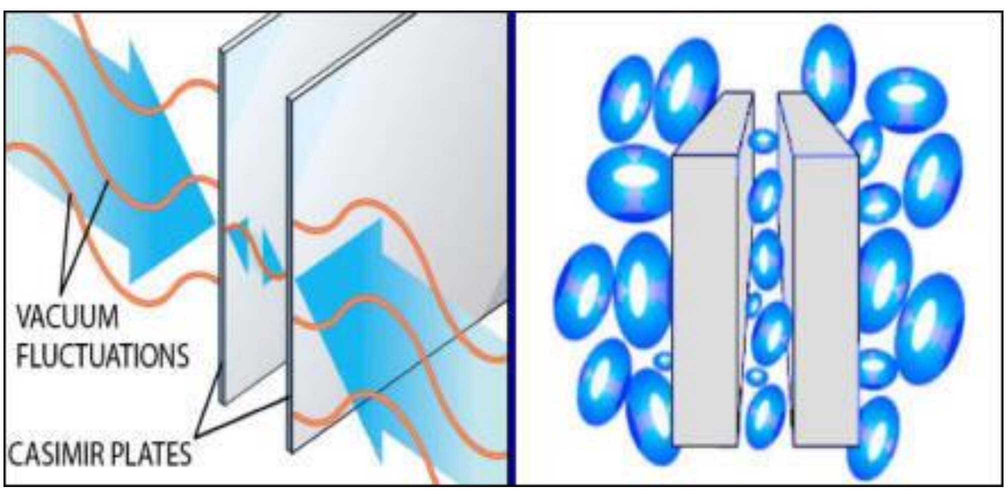
A crucial element in understanding the Casimir effect is understanding the environment in which it takes place. In a vacuum where there is nothing else, electromagnetic fields and fluctuations of those fields still exist. The idea of fluctuations comes from the Heisenberg uncertainty principle. Based on the uncertainty principle, photons come into existence for a small period of time, "borrowing" energy for that time and "giving it back" once they cease to exist again. During that existence, the photons "produce a tiny shift in the energy levels of atoms" (Braungardt). The tiny shifts create waves and fluctuations that interact with the things in the vacuum. The fluctuations are also called zero-point oscillations. The Casimir effect is caused by the fluctuations, so it is classified as a quantam electrodynamic effect (QED) (Levin 3).

The basic idea behind the Casimir effect begins with two tiny parallel mirrors in a vacuum, usually less than 100 nanometers apart. That separation is when the Casimir force starts to become significant (Harvard). Although that separation seems tiny, the Casimir effect is known as a “long-range effect” because the separation of the mirrors is large enough that the time it takes for a photon to travel from one mirror to the other and back again is “larger than some relevant characteristic period of either” mirror (Levin 2).

The aforementioned vacuum fluctuations travel through the mirrors and are reflected back and forth between them. If the separation between the mirrors is such that “integer multiples of half a wavelength [of the vacuum fluctuations] can fit exactly” it is said to be at a “cavity resonance frequency” (Lambrecht). When the fluctuations are at cavity resonance frequency the field inside of the mirrors is increased in magnitude but if they cannot, then the field is decreased. If the field between the mirrors is increased, then the energy of the field is increased and the pressure of the field is also increased. This makes it greater than the pressure outside of the mirrors, which remains the same. This pressure is called “field radiation pressure.” It is this difference in pressure that causes the mirrors to be pushed together or pushed apart (Figure 1). It is more likely for the wavelengths of the vacuum fluctuations to not be at cavity resonance frequency than for them to be at cavity resonance frequency. So, usually the force pushing the mirrors together is greater than the force pushing them apart, and the Casimir force is generally an attractive one (Lambrecht), but as recent exploration has shown, there are ways of making it a repulsive force.

The zero-point energy, or Casimir energy, of the two plates is given by $\varepsilon(a) = -\frac{\pi}{24a}$. Since $F = -\partial\varepsilon/\partial a$, the value of the Casimir force between two ideal mirrors is $-\pi/24a^2$. Also the force per unit area of an ideal mirror created by the Casimir force is given by the equation $F = -\pi^2/240a^4$ where a is the separation of the plates (Mostepanenko 6). Another form of the equation for the Casimir force is $F = -(\pi^2/240)(hc/a^4)$The lack of other variables is indicative of the interesting fact that the Casimir force does not depend on mass or other things.

If walls are thought of as many atoms, instead of a solid wall, the calculations change a bit. In this instance, the Casimir force is thought of as a van der Waal force occurring between many atoms instead of two. The van der Waals interaction is represented by $-C_7/R^7$. Therefore, the force per unit area changes to $F/A \propto -C_7 n_1 n_2/z^4$ where C_6 is a constant with the dimensions $(force)x(length)^7$, n_i is the number of atoms per unit volume in the *i*th wall, and z^4 is separation of the walls (Levin 18).



(Figure 1) http://physicaplus.org.il/zope/home/en/1223032001/ulf_en/image006.jpg

The equations associated with the Casimir effect are not exactly accurate because they designed for ideal mirrors in vacuums at a temperature of absolute zero. Ideal mirrors do not exist and absolute zero has been unattainable on Earth.

Real mirrors differ from ideal mirrors in that they are not perfectly smooth, and so they do not perfectly reflect the vacuum fluctuations. Also the change in separation produced by the roughness can have a large effect on the magnitude of the Casimir force since it is cut to about one sixteenth of its previous value when the separation is doubled. Real mirrors average a roughness around fifty nanometers (Lambrecht), which does not sound like much and is not even visible to the naked eye. But when the separation of the mirrors is only one hundred nanometers, the roughness causes a fifty percent change in the separation, which makes quite a bit of difference in the magnitude of the Casimir force.

Also, the characteristics of metallic materials have to be taken into account when calculating the Casimir force of two real parallel mirrors. At a distance of 0.1μm, the actual Casimir force is only half of the predicted Casimir force that two perfect mirrors would experience (Lambrecht). Each metal has a characteristic, δ, which is the "penetration depth of the field into the metal" (Mostepanenko 63). Using this, the Casimir force, F, between two plates of a metal with field penetration depth δ and separation *a* is given by the equation $F = -\frac{\pi^2}{240a^4}\left(1 + \frac{11}{3}\frac{\delta}{a}\right)^{-16/11}$ (Mostepanenko 64). The correction can also be made using the conductivity, σ, of the metal instead. The equation for that is $F = -\frac{\pi^2}{240a^4}\left(1 - \frac{1.93}{\sqrt{a\sigma}}\right)$, but it is only accurate at absolute zero (Mostepanenko 65).

At temperatures above absolute zero, thermal fluctuations along with the already present vacuum fluctuations come into play, adding to the pressure outside of the mirrors and increasing the Casimir force (Lambrecht). Although the correction in calculations brought on by the difference in actual temperature from the theoretical temperature of absolute zero are fairly small at the distance experiments are usually performed, the rate at which the correction grows is pretty large. At a separation of about 1 micrometer and a temperature of about 300 Kelvin, the correction is about 0.3%. At a separation of about 6 micrometers and a temperature of about 30 Kelvin, the correction is about 2%. If the temperature is increased by a factor of two, the correction increases by a factor of 16, or $\Delta_T F \propto (\Delta T)^4$ (Mostepanenko 95). However, at separations below one micrometer, thermal fluctuations have little effect on the experiment being performed since the wavelengths of the thermal fluctuations are too large to fit between the two mirrors (Lambrecht). This is good because, as mentioned above, the Casimir force is not thought to be significant until the separation is less than one hundred nanometers.

Another thing that makes measuring and experimenting with the Casimir force complicated is the fact that the mirrors have to be perfectly parallel with one another. This is not easily achieved. Since the Casimir effect is easily affected by differences in separation (Lambrecht) and the plates would be separated by different distances at each point, the plates would experience a different magnitude of Casimir force at each point. Even when the experiment was performed with two parallel plates in this decade, the error was 15% of the theoretical value (Lambrecht). Attempts at removing this difficulty have been made by scientists who have used different set-ups for measuring the Casimir effect. One scientist used a sphere and a flat disk in their experiments, moving the sphere closer to the disk and measuring the Casimir force along the way. The error from this method was only 1% of the predicted value (Lambrecht). Geometrically, this makes sense. No matter how the sphere rotates, its closest point to the disk is always the same distance from the disk.

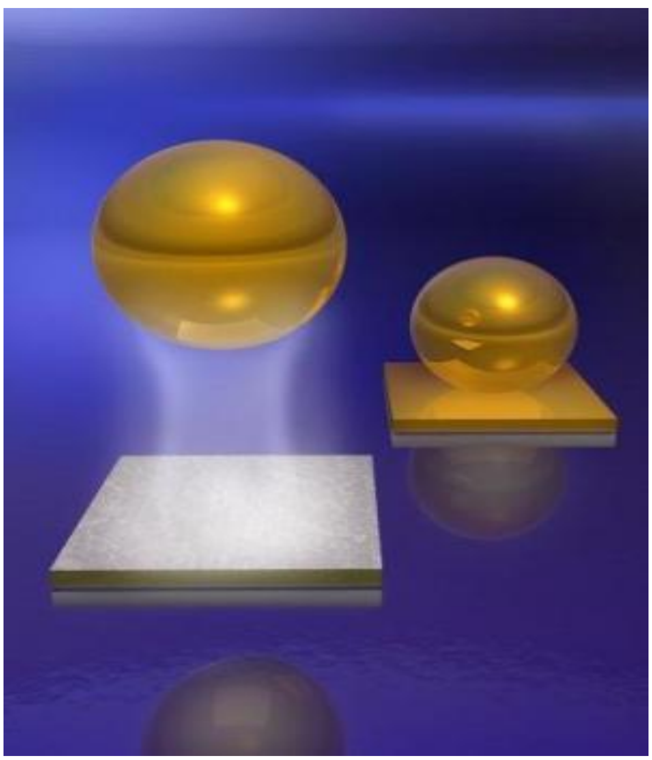
The Casimir effect can also occur between a variety of other shapes and materials, and the magnitude of its force can be found theoretically each time. The equation of the potential Casimir energy between two perfectly conducting spheres is $\varepsilon_S(a) = -\frac{143}{16\pi}\frac{R_1^3 R_2^3}{a^7}$ (Mostepanenko 85), where a is the separation of the spheres and R_n^3 represents the radii of the two spheres. For a perfectly conducting sphere close to a plane, the formula for the Casimir energy between the two objects is is $\varepsilon_{SP}(a) = -\frac{9}{16\pi}\frac{R^3}{a^4}$

The equations for the Casimir energy of two objects of any shape are surprisingly accurate. Using the simple equation $\varepsilon_S(a) = -\frac{2\pi^3}{27}\frac{R_1^3 R_2^3}{a^7}$ to predict the Casimir energy, the error cannot exceed 19.3% (Mostepanenko 85). When one of the objects is a plane, and the other is of any shape, the error is even smaller. The equation of the Casimir energy for this is $\varepsilon_{SP}(a) = -\frac{\pi^3}{180}\frac{R^3}{a^4}$ and it is accurate to a maximum error of 3.8% (Mostepanenko 86).

Since the Casimir force (F) is equal to $-\partial\varepsilon/\partial a$ it can be inferred that the Casimir force between two spheres is $F_S = \frac{-\partial\varepsilon(a)}{\partial a}\left(\frac{-143}{16\pi}\frac{R_1^3 R_2^3}{a^7}\right) = -\frac{1001}{16\pi}\frac{R_1^3 R_2^3}{a^8}$, a sphere and a plane is $F_{SP} = \frac{-\partial\varepsilon(a)}{\partial a}\left(-\frac{9}{16\pi}\frac{R^3}{a^4}\right) = -\frac{9}{4\pi}\frac{R^3}{a^5}$, two objects of any shape is $F_S = \frac{-\partial\varepsilon(a)}{\partial a}\left(-\frac{2\pi^3}{27}\frac{R_1^3 R_2^3}{a^7}\right) = -\frac{14\pi^3}{27}\frac{R_1^3 R_2^3}{a^8}$, and a plane and an object of any shape is $F_S = \frac{-\partial\varepsilon(a)}{\partial a}\left(-\frac{\pi^3}{180}\frac{R^3}{a^4}\right) = -\frac{\pi^3}{45}\frac{R^3}{a^5}$.

The Casimir force occurs between two metal mirrors covered in a dielectric material of thickness g, and its being covered in a dielectric material can even make the mirrors behave more like ideal mirrors. The Casimir force between two real metal mirrors is given by the equation $F = -\frac{\pi^2}{240a^4}\left(1 - \frac{16}{3}\frac{\delta}{a} + \frac{8}{3}\frac{g}{a}\right)$. When $g = 2\delta$, the two terms that change the original equation for the Casimir force are canceled out and the mirrors behave like perfect mirrors (Mostepanenko 70).

As long as the two objects are made of the same material, there will be an attractive Casimir force between them at certain distances (Harvard). But when two metallic objects of different materials are microscopic distances from each other, a repulsive Casimir force, and even levitation, can occur between them as illustrated in Figure 2. Figure 2 illustrates the levitation that occurs when a sphere is coated in gold and the plate is silica as opposed to the attraction that occurs when both the sphere and plates are coated in gold, an experiment performed by researchers at Harvard University (Harvard).



(Figure 2)*Courtesy of the lab of Federico Capasso, Harvard School of Engineering and Applied Sciences:* **http://www.sciencedaily.com/releases/2009/01/090107161422.htm**

This will certainly affect the way nanotechnology is designed as it evolves, since the attractive Casimir effect is responsible for many problems in nanotechnology. The Casimir force causes the small pieces to stick together, making the parts immovable. An example of this is in micro electromechanical systems (MEMS), such as devices that trigger air bags in vehicles (Leonhardt). Devices like this are prevented from being created smaller because the Casimir force would cause its parts to stick together and the device would be unable to function. In other instances, it is responsible for friction between parts (University of St. Andrews).

Another option for reversing the Casimir effect is placing a lens between the two plates. This lens would be in the form of a left-handed metamaterial. Left-handed metamaterials can be recognized by the fact that they refract objects' images opposite of how something like water would refract them (Figure 3) (Wegener). The lens between the plates transforms the attractive Casimir force that would be there into a repulsive Casimir force, and can cause stable levitation when the plates' normals are placed parallel to the force of gravity (Leonhardt).

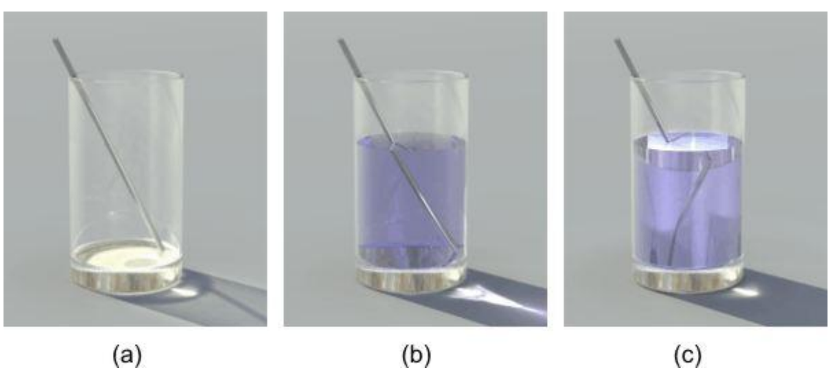


Figure 3 (a) shows an object, (b) shows that object in something that refracts normally, and (c) shows a left-handed metamaterial.

http://www.aph.uni-karlsruhe.de/wegener/en/research/metamaterials

The Casimir force is not always a negative aspect of nanotechnology and MEMS. It has been used to control the way that a MEMS device moves. In one instance, it was used to rotate a plate connected to a torsional rod. In another instance, the Casimir force was used to slow the oscillation of that same plate, making it easier to observe (Lambrecht). As research continues, more uses for the Casimir effect are sure to be found.

From its humble beginnings in mayonnaise to its prominent role in the development of levitation technology, the Casimir effect has remained an interesting mystery to physicists everywhere. This is no surprise since the Casimir effect is counterintuitive at first, raising eyebrows and increasing interest. Then after further investigation it encompasses many aspects of physics. Although the measurement of the Casimir force is complicated, the prediction of its magnitude is surprisingly accurate. The Casimir effect, although small and seemingly unimportant to people in general, is rising in importance as the demand for smaller and more efficient technology increases.

Bibliography

Braungardt , Jürgen “How the Universe Came from Nothing” http://www.braungardt.com/Physics/Vacuum%20Fluctuation.htm

Harvard University. "Levitation At Microscopic Scale Could Lead To Nanomechanical Devices Based On Quantum Levitation." ScienceDaily 7 January 2009. 11 April 2009 <http://www.sciencedaily.com /releases/2009/01/090107161422.htm>.

Lambrecht, Astrid. “The Casimir effect: a force from nothing.” Sept 1, 2002. <http://physicsworld.com/cws/article/print/9747>

Leonhardt, Ulf. “Quantum Levitation”. http://www.st-andrews.ac.uk/~ulf/levitation.html

Levin, Frank S., and David A. Micha. Long-Range Casimir Forces . New York: Plenum Press, 1993.

Mostepanenko, V.M., and N.N. Trunov. The Casimir Effect and its Applicants. New York: Oxford University Press, Inc., 1997.

University of St Andrews. "New Way To Levitate Objects Discovered." ScienceDaily 6 August 2007. 10 April 2009 <http://www.sciencedaily.com /releases/2007/08/070806091137.htm>.

Wegener, Martin. Metamaterials: Introduction. http://www.aph.uni-karlsruhe.de/wegener/en/research/metamaterials