

Lecture 4 1/22/03

This lecture introduces dynamics in one-dimension.

Section 4.1 Overview

~~Rectilinear~~ Motion = Motion in a straight line.

Newton's Laws

II. Defn Force - Time rate of change of momentum

$$F \equiv \frac{dp}{dt}$$

I. Isolated object experiences no force.

III. Momentum is conserved in each interaction pair

$$\Delta p_{12} + \Delta p_{21} = 0$$

$$\Rightarrow \frac{\Delta p_{12}}{\Delta t} + \frac{\Delta p_{21}}{\Delta t} = 0$$

$$F_{12} = -F_{21}$$

What are we doing?

Solve $F = \frac{dp}{dt} = m \ddot{x} = m \ddot{x}$ (mass constant)

for

Trajectory - $x(t)$
Velocity $v(t)$ or $v(x)$

Turning Points (x_t) $\dot{x} = 0$
Terminal Velocity (v_t) $\ddot{x} = 0$

What can we find out from trajectory?

$$\text{Work Done on Particle} - W = \int_{x_i \rightarrow x_f} F dx$$

Rate Work is Done (Power)

$$\begin{aligned} P &= \frac{dW}{dt} = \frac{d}{dt} \int_{x_i}^x F dx \\ &= \frac{dx}{dt} \frac{d}{dx} \int_{x_i}^x F dx = v F \end{aligned}$$

$$\text{Impulse } \Delta p = \int_{t_i}^{t_f} F dt$$

Rate momentum is transferred

$$\frac{d\Delta p}{dt} = \frac{d}{dt} \int F dt = F$$

Different Ways to Write $F=ma$

$$a = \ddot{x} = \dot{v} = m \frac{dv}{dx} \frac{dx}{dt} \quad (1)$$

$$= m v \frac{dv}{dx} \quad (2)$$

$$= \frac{1}{2} m \frac{dv^2}{dx} \quad (3)$$

$$\begin{aligned} W &= \int F dx = \int \frac{1}{2} m \frac{dv^2}{dx} dx \\ &= \int \frac{1}{2} m dv^2 = T_f - T_i \end{aligned}$$

change in kinetic energy.

Section 4.2 - Potential

The force can be separated into a reversible and non-reversible parts

$$F = F_{\text{rev}} + F_{\text{non}}$$

Reversible ~~forces~~ (conservative) Forces - When system is returned to original configuration, universe returns to original configuration.
 \Rightarrow State of system independent of trajectory.

Potential Energy - $V(x)$

$$F_{\text{rev}} = - \frac{dV}{dx}$$

Example F_g is conservative

$$F_g = -mg$$

$$V(x) = mgx$$

$$F_g = - \frac{dV}{dx} = -mg$$

$$\begin{aligned}
 T(x) - T(x_i) &= \int_{x_i}^x F_{ax} \\
 &= \int_{x_i}^x -\frac{dV}{dx} dx + \int F_{non} dx \\
 &= -V(x) + V(x_i) + \int F_{non} dx
 \end{aligned}$$

$$E = T + V = T(x_i) + V(x_i) + \int F_{non} dx$$

$\int F_{non} dx$
 Energy provided to
 or dissipated
 by system.

Section 4.3 - Viscous Forces

The chapter introduces a new kind of force, ~~the~~ air drag

$$F_{\text{drag}} = C_1 v + C_2 v^2$$

$$C_1 = 1.55 \times 10^{-4} D$$

$$C_2 = 0.22 D^2$$

} arbitrary units

$D \equiv$ Diameter.

Caveats

- * For smooth spheres
- * Fluid Flow is complex
- * Depends on fluid density.