Lecture 5 - 1/25/03

This lecture introducees potential and continues to work rectalinear problems.

Section 5.1 - Homework

I used the following math all the time on the last homework set.

(1) $\vec{v}\cdot\vec{v} = v^2 =$

(2) $v = \sqrt{v_x^2 + v_y^2 + v_z^2}$

(3) $\frac{d}{dt}\vec{v}_1\cdot\vec{v}_2 = \vec{v}_1\cdot\frac{d\vec{v}_2}{dt} + \cancel{\vec{v}_2\cdot\frac{dv_1}{dt}}$

$\left(\frac{dv_1}{dt}\right)\cdot v_2$

(4) $\vec{v}_1\cdot\vec{v}_2 = \vec{v}_2\cdot\vec{v}_1$

(5) v, a in cylindrical + spherical

(6) $\vec{A}\cdot\vec{B} = 0$ if $\perp$ $\quad \vec{A}\cdot\vec{B} = AB$ if $\parallel$

$\boxed{\text{I will help with math on first test}}$

Hockey Pucks

Approximation I - Earth fixed. (Good dynamics)

$\vec{L}$, E conserved. P not conserved

Approximation II - Earth point charge

$\vec{L}$, $\vec{E}$, $\vec{p}$ conserved, but torque that isn't balanced.

Approximation III - Earth sphere, conserve everything but what a mess.

Section 5.1 - Potential

Forces that depend only on x can be expressed in terms of a potential.

$$F(x) = -\frac{dU}{dx}$$

$$T(x) - T(x_0) = \int_{x_0}^{x} -\frac{dU}{dx}\, dx = U(x_0) - U(x)$$

Total Energy $\quad E = T + U = \text{const.}$

- $T = \frac{1}{2} m v^2$

Problem - Consider the force $F = -A \sin \omega x$ where a particle of mass m has kinetic energy T_0 at $t=0$ $x=0$. ~~Solve for the trajectory.~~

(a) Find the turning points and a condition the particle must satisfy to be bound.

(b) Solve for the velocity of the particle as a function of time.

(c) Solve for the trajectory.

Solve for the potential

$$F = -A \sin \omega x = -\frac{dU}{dx}$$

$$\int A \sin \omega x = \int dU$$

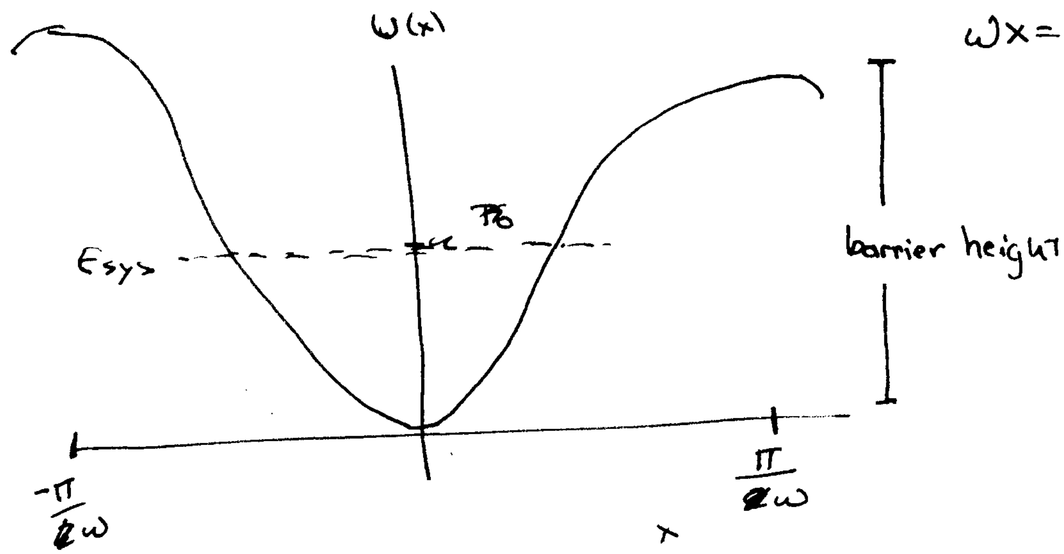
$$-\frac{A}{\omega} \cos \omega x = U(x) + C$$

Select $U(0) = 0$

$$U(x) = \frac{A}{\omega}\left(1 - \cos \omega x\right)$$

$\omega = \cancel{\frac{2\pi}{T}}$

$\omega x = \frac{\pi}{\cancel{2}}$



$$U_{max} = \frac{2A}{\omega}$$

The condition to be bound is $E < U_{max}$

The total energy of the system is $E = T_0$ since $U(0) = 0$.

So to be bound, $T_0 < U_{max} = \frac{2A}{\omega}$

(b) The pariticle turns when $v = 0$
$\Rightarrow T = 0$

$$T_0 = E_{sys} = U(x) = \frac{A}{\omega}\left(1 - \cos\omega x\right)$$

$$\cancel{\frac{A}{\omega}\ \ T_0} =$$

$$1 - \frac{\omega T_0}{A} = \cos\omega x$$

$$x = \cos^{-1}\left[1 - \frac{\omega T_0}{A}\right]$$

Note when $T_0 = \frac{2A}{\omega}$

$$x = \cos^{-1}\left[1 - 2\right] = \cos^{-1}\left[-1\right]$$

if T_0 is larger, this can't be solve.

(c) Solve for v as a function of time –

$$E_{sys} = T_0 = T + U$$

$$= \frac{1}{2}mv^2 + \frac{A}{\omega}\left[1 - \cos\omega x\right]$$

$$T_0 - \frac{A}{\omega}\left[1 - \cos\omega x\right] = \frac{1}{2} m v^2$$

$$\frac{2}{m}\left[T_0 - \frac{A}{\omega} + \frac{A}{\omega}\cos\omega x\right] = v^2$$

Let $\cancel{a = \frac{2}{m}\left(T_0 - \frac{A}{\omega}\right)}$

$$\frac{2A}{m\omega}\left[\underbrace{\frac{T_0\omega}{A} - 1}_{} + \cos\omega x\right] = v^2$$

$$v(x) = \sqrt{\frac{2A}{m\omega}\left(a + \cos\omega x\right)}$$

where $a = \dfrac{T_0\omega}{A} - 1$

Turning Points ensure $\sqrt{\text{positive}}$

(a) <u>Trajectory</u>

$$\frac{dx}{dt} = v = \sqrt{\frac{2A}{m\omega}\left(a + \cos\omega x\right)}$$

$$\int_0^x \frac{dx}{\sqrt{\frac{2A}{m\omega}\left(a + \cos\omega x\right)}} = \int dt = t$$

$$\int_0^x \frac{dx}{\sqrt{a + \cos\omega x}} = \left(\sqrt{\frac{2A}{m\omega}}\right) t$$

Math Hand book

$$\int_0^x \frac{dx}{\sqrt{a + b\cos x}} = \frac{2}{\sqrt{a+b}} F\left(\frac{x}{2}, r\right)$$

$$r = \sqrt{\frac{2b}{a+b}}$$

So you were done, if you get an integral you can't do ask in class.

F = Complete elliptic integral of first kind

Section 5.4

Suppose $T_0 \ll E_{sys}$

Expand the potential about equilibrium,

$$U(x) = U(\cancel{x}_0) + \left.\frac{dU}{dx}\right|_{x_0}(x-x_0) + \frac{1}{2}\left.\frac{d^2U}{dx^2}\right|_{x}(x-x_0)^2$$

$$x_0 = 0$$

$$U(x_0) = 0$$

$$\left.\frac{dU}{dx}\right|_{x_0} = \cancel{0}\ \frac{d}{dx}\frac{A}{\omega}\left[1 - \cos\omega x\right] = -F$$

$$= \frac{A}{\cancel{\omega}} \sin\omega x \Big|_0 = 0$$

$\Rightarrow$ Good Force at equilibrium better be zero.

$$\frac{d^2U}{dx^2} = A\omega\cos\omega x \Big|_0 = A\omega$$

$$U(x) \approx \frac{A\omega}{2} x^2$$

$\Rightarrow$ Reduced the problem to a simple harmonic oscillation $k = A\omega$

Section 5.5

Playball blown without rolling is regime where quadratic air drag dominates.

$$F = c_2 (v - v_w)^2$$

Method I

Use $F = m\dfrac{dv}{dt} = c_2 (v - v_w)^2$

$$\int_0^v \frac{dv}{(v - v_w)^2} = \int_0^t \frac{c_2\,dt}{m}$$

Method II

$$F = m\frac{dv}{dx}\frac{dx}{dt} = mv\frac{dv}{dx} = c_2 (v - v_w)^2$$

$$\frac{c_2 x}{m} = \int_0^x \frac{c_2}{m}dx = \int \frac{v\,dv}{(v - v_w)^2}$$

$$=$$

Section 4.4 - Example

A playball is blown over a frictionless icy surface by a wind with velocity V_w. Assume quadratic term dominates in drag. Compute everything

$$F = c_2 (v - V_w)^2$$

(1) Terminal velocity - $F = 0 \Rightarrow v_t = V_w$

(2) $v(x)$

$$F = m v \frac{dv}{dx} = c_2 (v - V_w)^2$$

$$\int_0^x dx = x = \frac{m}{c_2} \int \frac{v\, dv}{(v - V_w)^2}$$

$$u = v - V_w \qquad du = dv$$

$$\frac{c_2 x}{m} = \int_{u_0}^{u} \frac{(u + V_w)\, du}{u^2} = \int_{u_0}^{u} \frac{du}{u} + \int_{u_0}^{u} \frac{V_w\, du}{u^2}$$

$$= \ln\left(\frac{u}{u_0}\right) - V_w \left[\frac{1}{u}\right]_{u_0}^{u}$$

$$= \ln\left(\frac{u}{u_0}\right) + \frac{V_w}{u_0} - \frac{V_w}{u}$$

$$= \ln\left(\frac{v - V_w}{-V_w}\right) + \frac{V_w}{-V_w} - \frac{V_w}{v - V_w}$$

Method 2

$$F = m \frac{dv}{dt} = c_2 (v - v_w)^2$$

$$\frac{c_2}{m} \int_0^t dt = \frac{c_2}{m} t = \int_0^v \frac{dv}{(v - v_w)^2}$$

$$u = v - v_w \qquad du = dv$$

$$\frac{c_2}{m} t = \int_{u_0}^{u} \frac{du}{u^2} = - \frac{1}{u} \Big|_{u_0}^{u}$$

$$= \frac{1}{u_0} - \frac{1}{u} = \frac{1}{-v_w} - \frac{1}{v - v_w}$$

$$= - \frac{1}{v_w} \left[1 - \frac{1}{1 - v/v_w} \right]$$

$$1 + \frac{v_w c_2 t}{m} = \frac{1}{1 - v/v_w}$$

$$\frac{1}{1 + \frac{v_w c_2 t}{m}} = 1 - v/v_w$$

$$v = v_w \left[1 - \frac{1}{1 + \frac{v_w c_2 t}{m}} \right]$$