Lecture 8 - Review for Exam 1

2/3/2003

Section 8.1 – Studying

Exam 1 – 6:00 pm Thursday Feb 6 in Chem 113

Covers Chap 1, 2, 3.1 – 3.4, 3.6

⇒ Problem 3.19 – 3.24 not covered.

How to Study?

I. Re-write you notes until you understand where the pieces come from. You should be able to derive each piece you use.

II. Work additional problems.

III. Invent your own problems, what could I ask.

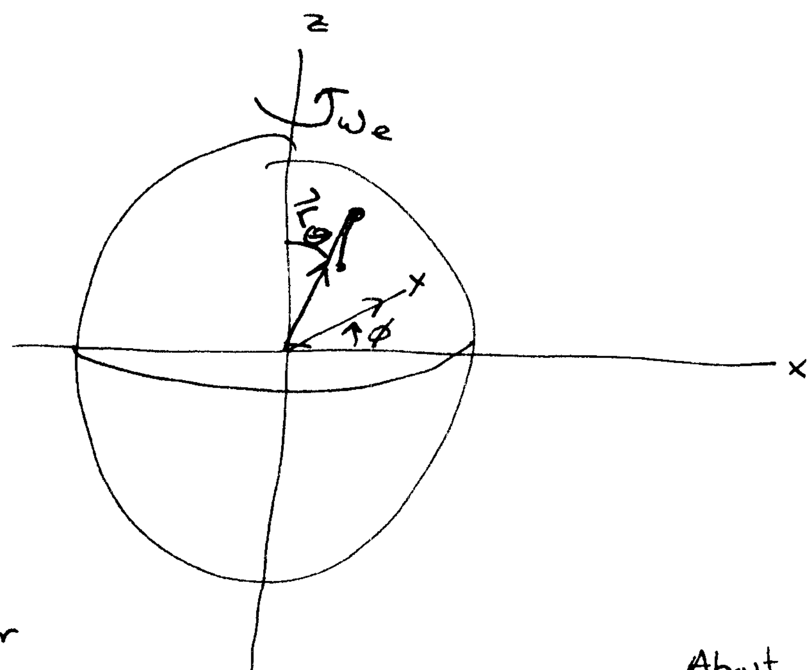
IV. Grab a chalk board and chalk it up

Section 8.2 - The trip south.

Problem - Using a rotating earth as the coordinate system compute, the $\vec{v}$, $\vec{a}$, KE, PE of a car travelling 70 mph southward.

$$\omega_e \equiv \frac{2\pi}{1\,\text{day}} \quad \text{angular velocity of Earth}$$

$$\omega_c \equiv \frac{V}{R_{earth}} = \text{angular velocity of car}$$



$$\vec{r} = R_e \hat{e}_r$$

$$\dot{r} = \ddot{r} = 0$$

From z axis

$$\theta(t) = \theta_F + \omega_c t$$

$$\dot{\theta} = \omega_c$$

$$\ddot{\theta} = 0$$

About z axis

$$\phi(t) = \omega_e t$$

$$\dot{\phi} = \omega_e$$

$$\ddot{\phi} = 0$$

$\boxed{1.12.12}$

$$\vec{v}(t) = \dot{r}\,\hat{e}_r + r\dot{\phi}\sin\theta\,\hat{e}_\phi + r\dot{\theta}\,\hat{e}_\theta$$

$$= 0 + R_e\omega_e\sin(\theta_F+\omega_c t)\,\hat{e}_\phi + R_e\omega_c\,\hat{e}_\theta$$

$$T = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m\vec{v}\cdot\vec{v} = R_e^2\omega_e^2\sin^2(\theta_F+\omega_c t) + R_e^2\omega_c^2$$

Note: KE not constant

$$U = -\frac{MmG}{R_e}$$

$$\vec{a}(t) = (\underset{0}{\ddot{r}} - r\dot{\phi}^2\sin^2\theta - r\dot{\theta}^2)\,\hat{e}_r +$$

$$(r\underset{0}{\ddot{\theta}} + 2\underset{0}{\dot{r}}\dot{\theta} - r\dot{\phi}^2\sin\theta\cos\theta)\,\hat{e}_\theta +$$

$$(r\underset{0}{\ddot{\phi}}\sin\theta + 2\dot{r}\underset{0}{\dot{\phi}}\sin\theta + 2r\dot{\theta}\dot{\phi}\cos\theta)\,\hat{e}_\phi$$

$$= \left[-R_e\omega_e^2\sin(\theta_F+\omega_c t) - R_e\omega_c^2\right]\hat{e}_r +$$

$$\left[-R_e\omega_e^2\sin(\theta_F+\omega_c t)\cos(\theta_F+\omega_c t)\right]\hat{e}_\theta +$$

$$\left[2R_e\omega_c\omega_e\cos(\theta_F+\omega_c t)\right]\hat{e}_\phi$$

Section 8.3 – ~~Problem~~ Dissipation

Problem A particle is ~~released~~ launched with velocity v_0 from point $b > 0$. The particle moves under a force

$$F = -\alpha x v \qquad \alpha > 0 .$$

(a) Is the force dissipative?

(b) Compute trajectory

(c) Compute every thing else.

(a) Force is velocity dependent and therefore dissipative

(b)

$$F = m\ddot{x} = mv\frac{dv}{dx} = -\alpha x v$$

$$\frac{mv}{v}\, dv = -\alpha x\, dx$$

$$m\int_{v_0}^{v} dv = -\alpha \int_b^x x^2 dx =$$

$$m(v - v_0) = \frac{-\alpha}{2}(x^2 - b^2)$$

$$v(x) = -\frac{\alpha}{2m}(x^2 - b^2) + v_0$$

(c) Turning Point

$$v(x) = 0 = -\frac{\alpha}{2m}(x^2 - b^2) + v_0$$

$$x_t = + \sqrt{\frac{2 m v_0}{\alpha} + b^2} \qquad \text{(stopping point)}$$

(c)

$$v(x) = -\frac{\alpha}{2m}\left(x^2 - b^2 - \frac{2 m v_0}{\alpha}\right)$$

$$= -\frac{\alpha}{2m}\left(x^2 - x_t^2\right) = \frac{dx}{dt}$$

$$-\frac{2m}{\alpha}\int_0^t dt = \int_{b}^{x} \frac{dx}{x^2 - x_t^2}$$

Let $u = x/x_t \qquad du = \dfrac{dx}{x_t}$

$$-\frac{2m}{\alpha}\, t = \frac{x_t}{x_t^2}\int_{u_0}^{u} \frac{du}{u^2 - 1} \qquad \cancel{t}$$

$$\boxed{\underline{\text{Integral}} \qquad \int \frac{du}{u^2-1} = -\int \frac{du}{1-u^2} = -\frac{1}{2}\ln\left(\frac{1+u}{1-u}\right)}$$

$$\frac{4 m x_t t}{\alpha} = \ln\left(\frac{1+u}{1-u}\right)\Bigg|_{b/x_t}^{x/x_t}$$

$$= \ln\left(\frac{1 + x/x_t}{1 - x/x_t}\right) - \ln\left(\frac{1 + b/x_t}{1 - b/x_t}\right)$$

$$-\frac{4mx_t t}{\alpha} = \ln\left[\left(\frac{1+b/x_t}{1-b/x_t}\right)\left(\frac{1-x/x_t}{1+x/x_t}\right)\right]$$

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(d) How much energy is lost to the system when it reaches turning (stopping point)

Method I $\quad \int_b^{x_t} dW = \int_b^{x_t} F\,dx$ $\quad$ Integrate Work with respect to distance

$$= -\int_b^{x_t} \cancel{\alpha}\, x v\, dx$$

$$= \frac{\alpha^2}{2m}\int_b^{x_t}(x^2 - x_t^2)\, x\, dx$$

Method II $\quad$ Integrate Power with respect to time.

$$\frac{dE}{dt} = Fv$$

$$\Delta E = \int_0^t \underbrace{\alpha x(t) v(t)}_{F} v(t)\, dt \qquad \text{OUCH}$$

Method III - The particle comes to a stop at the turning point. There is no potential energy it much have lost all its energy $\Delta E = \frac{1}{2} m v_0^2$

Lecture 9 - Review for Test 9

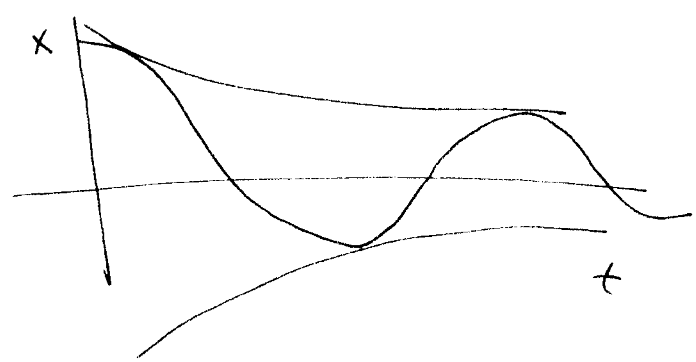
- Test Chem 113 Tomorrow, 6:00 pm.
  No time limit. One formula sheet.
  Bring all math book you want. You
  will work on your own paper.

Section 9.2 - Oscillations Review

4 frequencies

ω_0 - Natural Frequency - You hit the system with no friction. This is the frequency it oscillates at.

ω_d - Damping Frequency - You hit an oscillator with friction, where $\omega_0 > \gamma$ so it does oscillate. This is the frequency it does oscillate at.



ω - Driving Frequency - This is an external frequency, has nothing to do with oscillator properties.

ω_r - Resonant Frequency - This is the driving frequency which produces the maximum amplitude response.

Trajectories

~~Over~~ damped

$$q = \sqrt{\gamma^2 - \omega_0^2}$$

$$x(t) = A_1 e^{-(\gamma+q)t} + A_2 e^{-(\gamma-q)t}$$

Critically Damped

$$q = 0$$

$$x(t) = A_1 e^{-\gamma t} + A_2 t e^{-\gamma t}$$

Over damped

$$q = i\omega_d$$

$$x(t) = A_4 \cos(\omega_d t + \phi_0) e^{-\gamma t}$$

Driven (Assume Under ~~Over~~ damped)

$$x(t) = \underbrace{A \cos(\omega_d t + \phi_0) e^{-\gamma t}}_{\text{Transient}} + \underbrace{A(\omega)\cos(\omega t + \phi_1)}_{\text{Steady State Response}}$$

$$A(\omega) = \frac{F_0/\omega}{((\omega^2 - \omega_0^2)^2 + 4\omega_4^2\gamma^2)^{1/2}} \qquad \tan\phi_1 =$$

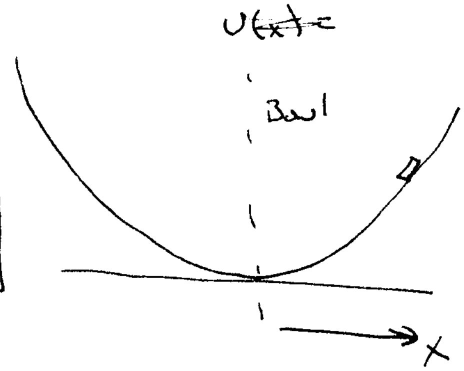
Relation between period T, frequency f, and angular frequency (velocity), ω

$$T = \frac{1}{f} \qquad 2\pi f = \omega$$

$$T = \frac{2\pi}{\omega}$$

Section 8.3 - Damped Oscillator

Problem I release a checker a distance x_0 from the center of a parabolic bowl $y(x) = \alpha x^2$. It requires $\Delta t_1 = 1.5s$ to complete its first cycle. Compute the trajectory, energy, rate of energy loss, etc. The second maximum is 3/4 the first max.



Is motion under/over/or critically damped? what would you see otherwise?

Sln Assume Linear damping

$$F = -\frac{dU}{dx} - cv$$

$$U(x) = mgy = mg\alpha x^2$$

$$F = -2mg\alpha x - cv = m\ddot{x}$$

$$m\ddot{x} + cv + 2mg\alpha x = 0$$

$$\ddot{x} + \frac{c}{m}\dot{x} + 2g\alpha x = 0$$

$$\gamma = \frac{c}{2m}, \quad \omega_0^2 = 2g\alpha$$

Underdamped $\quad \gamma < \omega_0 \quad \omega_d^2 = \omega_0^2 - \gamma^2$

$$\frac{A(t_2)}{A(t_1)} = e^{-\gamma T_d} \qquad T_d = 1.5\,s$$

$$\frac{3}{4} = e^{-\gamma T_d}$$

$$\ln(3/4) = -\gamma T_d$$

$$\gamma = -\frac{1}{T_d}\ln(3/4) = 0.19\,s^{-1}$$

$$\omega_d = \frac{2\pi}{T_d} = 4.19\,s^{-1}$$

$$\omega_0 = \sqrt{\omega_d^2 + \gamma^2} = 4.19\,s^{-1}$$

$\omega_0 \gg \gamma \qquad \omega_d \approx \omega_0 \qquad$ Very weakly damped

Solve for α to find the shape of my bowl.

$$\omega_0^2 = 2g\alpha \qquad \alpha = \frac{\omega_0^2}{2g} = 0.89\,m^{-1}$$

So if bowl has radius 10cm, hieght at end 0.9cm

$\Rightarrow$ very slippery bowl.

<u>Trajectory</u> $\quad x(0) = x_0$

$\dot{x}(0) = 0$

$$\left.\begin{aligned} x(t) &= A_1 e^{-\gamma t}\cos\omega_d t + A_2 e^{-\gamma t}\sin\omega_d t \\ &= A e^{-\gamma t}\cos(\omega_d t + \phi) \end{aligned}\right] \text{equivalent}$$

use

$$x(0) = A_1 = 0$$

$$\dot{x}(t) = -A_1\gamma e^{-\gamma t}\cos\omega_d t - A_1 e^{-\gamma t}\omega_d \sin\omega_d t$$
$$- A_2\gamma e^{-\gamma t}\sin\omega_d t + A_2 e^{-\gamma t}\omega_d \cos\omega_d t$$

$$\dot{x}(0) = -A_1\gamma + A_2\omega_d = 0$$

$$A_2 = \frac{A_1\gamma}{\omega_d} = \frac{x_0\gamma}{\omega_d}$$

<u>Energy as a function of time</u>

$$E = T + U = \tfrac{1}{2}mx^2 + mg\alpha x^2 \stackrel{?}{=} \text{constant}$$

<u>Rate of Energy Loss</u>

$$\dot{E} = P_{\text{dissipative force}} = F_{\text{diss}}\, v$$
$$= -cv^2$$

Section 9.4 - Driven Oscillations

Drive the same system with a periodic force $F = F_0 \cos \omega_0 t$

$\Rightarrow$ Maybe charge the object to Q and apply a time varying field

$F = Q E_0 \cos \omega t$

For a given frequency ω, there will be a trajectory

$$x(t) = A_1 e^{-\gamma t} \cos(\omega_d t + \phi_d) + A(\omega) \cos(\omega t + \phi_0)$$

$$A(\omega) = \frac{F_0/m}{\left((\omega_0^2 - \omega^2)^2 + 4\gamma^2\omega^2\right)^{1/2}}$$

$$\tan \phi_0 = \frac{2\gamma\omega}{\omega_0^2 - \omega^2}$$

Because of the $e^{-\gamma t}$ term the transients die away, and were left with the steady state solution.

Select a driving force with a 2s period.

$$T = 2s \qquad f = \frac{1}{2} s^{-1} \qquad \omega = 2\pi f = \pi s^{-1}$$

Let $F_0/m = 10 m/s^2$

Recall $\omega_0 = 4.19 s^{-1}$ so we are near resonance.

$$\omega_r^2 = \omega_0^2 - 2\gamma^2 = \omega_d^2 - \gamma^2 =$$

$$= (4.19 s^{-1})^2 - (0.19 s^{-1})^2 \approx (4.19 s^{-1})^2$$

$$\omega_r = 4.19 s^{-1}$$

$$A(\omega) = \frac{F_0/m}{((\omega_0^2 - \omega^2)^2 + 4\gamma^2\omega^2)^{1/2}}$$

$$= \frac{10 \text{ m/s}^{-1}}{((4.19)^2 - \pi^2)^2 + 4(0.19)^2(\pi)^2)^{1/2}} = 0.129 s^2 \cdot 10 \text{ m/s}^2$$

$$= 1.29 \text{ m}$$

$\tan \phi =$