Section 1.1.3 - Systems and Conservation Laws

System - Any collection of objects where one can clearly determine what objects are in the collection and whether interactions occur between objects within the system or between ~~ob~~ objects within the system and objects within the environment.

Environment - Everything not in a system.

In ~~An~~ an isolated system (the universe for example), the total energy, linear momentum, and angular momentum is conserved.

Energy - E_{sys} - Total Energy of System,
So for an isolated system

$$\Delta E_{sys} = 0$$

Mechanic Energy -

Kinetic Energy - $\sum \frac{1}{2} m v^2 = K_{sys} = \sum \frac{p^2}{2m}$

Potential Energy - U - The energy of the system stored within reversible determations of the system.

Source Energy - ΔE_{sourse} - Energy provide by some sourse, yw, battery, nuclear explosion, that cannot or we chose not to

Dissipated Energy - ΔE_{diss} - Energy converted into heat or other forms ~~not~~ that cannot be easily converted back into mechanical energy.

<u>Linear Momentum</u> $(\vec{p})$ - Total linear momentum of a system $\vec{P}_{sys} = \sum_i m_i \vec{v}_i$

<u>Angular Momentum</u> $(\vec{L})$ -

$$\vec{L}_{sys} = \sum_i \vec{r}_i \times \vec{p}_i$$

* $\vec{r}_i$ is the displacement from any of rotation. The angular momentum must be conserved about each point.

We have a set of three conservation laws for an isolated system. A large number of systems can be analysed using only these conservation laws. This should always be the first thing you try.

<u>Non-Isolated Systems</u> - Interactions between the system and the environment.

The key element in our definitions of energy, momentum, and angular momentum is $\vec{p}$. For the bookkeeping to work out and the momentum of the universe to be conserved, interactions must remove the same amount of momentum from the environment, that is added to a system.

<u>Force</u> - Time rate of change of momentum

$$\vec{F}_{net} = \frac{d\vec{p}}{dt} \qquad \text{* Newton 2}$$

Interaction Pair - $\vec{F}_{12} = -\vec{F}_{21}$

An interaction transfers energy, momentum, and angular momentum between systems.

Transfer of Momentum - Impulse

$$\Delta \vec{p}_{sys} = \int \sum \vec{F} \, dt$$

Transfer of Energy - Work

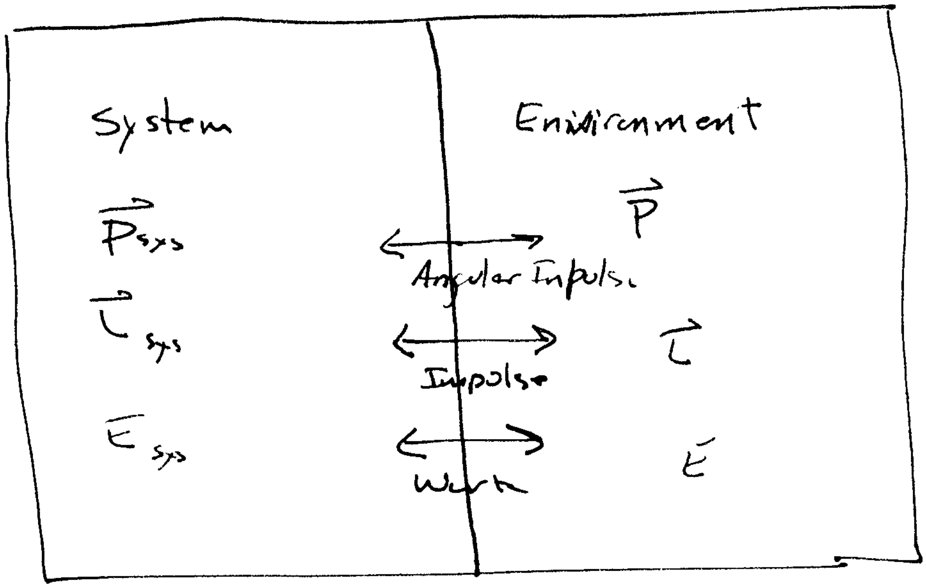
$$W = \Delta E_{sys} = \int \sum F_i \Delta x_i$$

Transfer of Angular Momentum - Angular Impulse?

$$\Delta L_{sys} = \int \sum \vec{\tau} \, dt$$

Torque (Tork) τ — $\vec{\tau} = \vec{r} \times \vec{F}$

Universe $\Delta E = \Delta L = \Delta p = 0$



Section 1.1.4 - Reference Frames

A reference frame is an orthogonal set of unit vectors e_1, e_2, e_3 and a point of origin.

Unit vectors - $e_i \cdot e_i = 1$ $\qquad e_i \times e_j = 0$ if $i \neq j$

Inertial Reference Frame - A frame where ~~the~~ Newton's First Law, ~~objects at rest stay at rest~~ holds.

$\Rightarrow$ Stationary ~~frames~~ or Frames moving at constant velocity are inertial frames.

Galelian Transformations

$$\vec{r}\,' = \vec{r} - \vec{v}t$$
$$t' = t$$

transform one inertial frame into another.

Problems - A Galelean Transformation changes both the momentum and kinetic energy, what about energy ~~transform~~ conservation?

A Galelean transformation leaves $\Delta\vec{v}$ and thus $\Delta\vec{p} \Rightarrow \vec{F} \Rightarrow W$ invariant so the conservation laws are maintained.

Section 1.1.5 - Forces

You already have quite a few forces that you know about and can deal with.

<u>Conservative Forces</u> - Forces that result in a reversible change in the system. The ~~[illegible]~~ change in the energy of system can be represented by a potential energy

~~[illegible]~~ <u>Field Forces</u> -

Electric ~~Gravitation~~ - $\vec{F}^e_{12} = \frac{\cancel{k}\, q_1 q_2}{4\pi\varepsilon_0 r_{12}^2} \hat{r}_{12}$

<u>Gravitation</u> - $\vec{F}^g_{12} = \frac{G M_1 m_2}{r_{12}^2} \hat{r}_{12}$

<u>Contact Forces</u>

<u>Linear Springs</u> $\vec{F}^h = -k(\vec{x} - \vec{x}_0)$

<u>Static Friction</u> - $F^s_{\cancel{s}} \leq \mu_s F^n$

<u>Normal Forces</u> (F^n)

Force of collision in an elastic collision

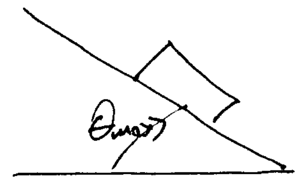
~~[illegible]~~

<u>Non-conservative Forces</u> -

<u>Kinetic Friction</u> (μ_K) $F^K_{12} = \mu_K F^n_{12}$

Force in inelastic collision.

Coefficient of Static Friction - μ_s - Does not depend on surface area of contact, found by the maximum angle before sliding.



$$\mu_s = \tan \theta_{max}$$

$$\mu_k < \mu_s$$

Lecture 2 – 1/15/2003

This lecture continues the review of mechanics

Section 2.1 - Diagrams

Free Body Diagram - Draw all forces on an object at the center of mass.

I. Draw a cartoon of the object. A dot is ok.

II. Draw each force on the object, but not net force.

III. Indicate direction of acceleration.

Extended Free Body Diagram

I. Draw axis of rotation.

II. Draw object in enough detail that location of forces can be identified.

III. Draw forces in plane of rotation.

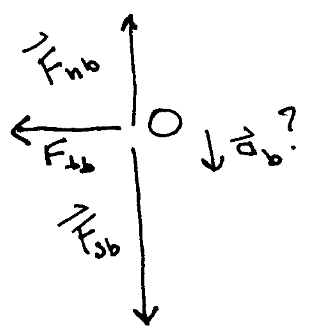
IV. Draw direction of rotational acceleration.

Example - Draw the free body and extended free body diagram of a baseball bat as it hits a baseball.

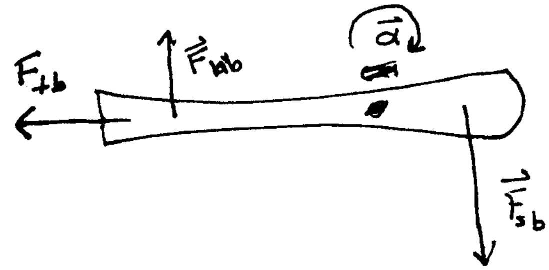
System - Baseball Bat

Environment - Everything Else.

Free Body Diagram



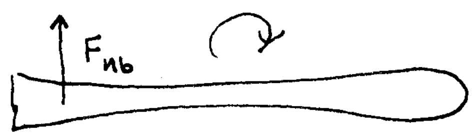
Extended Free Body Diagram



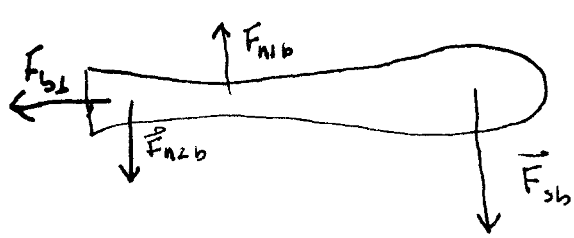
h ≡ hitter   b ≡ bat   s ≡ ball

⇒ Not free rotation so axis of rotation undetermined.

Ask - Is the picture correct, what is the motion of the bat ~~when~~ before striking the ball?



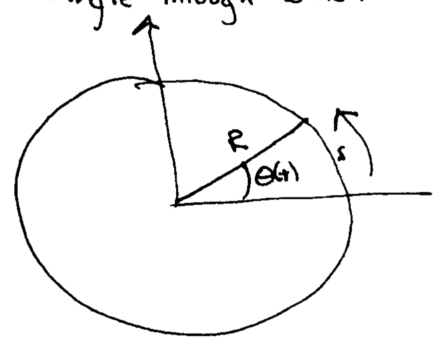
Correct model



Section 2.1 – Angular Momentum

~~A rolling disk~~ ~~The~~ The rotation of a disk about it center is described giving the angle through which the disk has turned as a function of time.

Angular Position $\theta(t)$ – ~~Angle through which~~



$$\theta(t) = \frac{s}{R}$$

where s is the distance a point on edge has traveled and R is the radius.

Angular Velocity – $\omega(t) = \dot{\theta}(t) = \frac{\dot{s}}{R} = \frac{v}{R}$

Angular Acceleration $\alpha(t) = \dot{\omega}(t) = \frac{\dot{v}}{R} = \frac{a}{R}$

The net torque about an axis is related to the angular acceleration by

$$\left|\sum \vec{N}\right| = \underbrace{\left|\sum \vec{r}\times\vec{F}\right|}_{\text{Torque}} = I\alpha$$

$\Rightarrow$ An object that experiences a net torque undergoes angular acceleration.

Section 2.2 - Analysing Systems

To analyze a system, you must

I. Define a coordinate system, identify system components

II. Draw Free Body and Extended Free Body Diagram for the system.

III. Identify internal and external interactions.

IV. Show how energy, momentum, and angular momentum are conserved in each interaction.

V. Based on your analysis, describe the approximate behavior of the system.

Example - Analyze throwing a baseball and flight using (A) the universe as the system, (B) The baseball

I. Use a coordinate system fixed to the earth, system components Earth (e), Thrower (t), Baseball (b)

~~II~~. The Universe

II. No external forces ⟹ no ~~free~~ diagrams

III. Interactions

I. ~~Earth~~ Throwing Baseball

II. Flight Baseball.

III. Landing Baseball.

~~Conservation~~ Throwing Baseball

	Before		After
Momentum	$-\ 0$	$=$	$\vec{P}_e^{If} + \vec{P}_b^{If}$
Energy	$-\ \cancel{0} E_t^{Ii}$	$=$	$\cancel{K}_e^{If} + \cancel{K}_b^{If} + E_t^{If}$

Flight of Baseball

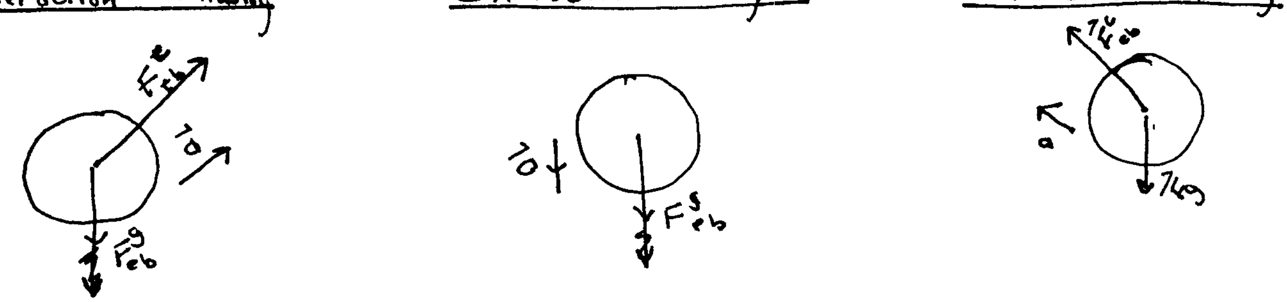
Interaction ~~between~~ Earth is extended so, everything is a function of time.

	Before		After
Momentum –	0	$=$	$\vec{P}_e^{II}(t) + \vec{P}_b^{II}(t)$
Energy –	$K_e^{If} + K_b^{If} + E_t^{If}$	$=$	$E_t^{II} + K_e^{II}(t) + K_b^{II}(t) + U_{eb}(t)$

Leave Landing to Student

Using Ball as System

Interaction Throwing   Interaction Flight   Interaction Landing



No extended diagram because were not analyzing angular momentum.

Throwing

$\vec{P}_b^{I,i} = 0$ $\vec{P}_b^{I,f} = \int \Sigma F \, dt$ = impulse of thrower + earth

$E_b^{I,i} = 0$ $K_b^{I,f} = \int \Sigma F \, dx_i$ = Work done on ball

Flight

$$\vec{P}_b^{II}(t) = \vec{P}_b^{I+} + \int \vec{F}_g \, dt$$

$$K_b^{II}(t) = K_b^{II+} + \int_{z_0}^{z(t)} \vec{F}_g \cdot d\vec{z}$$

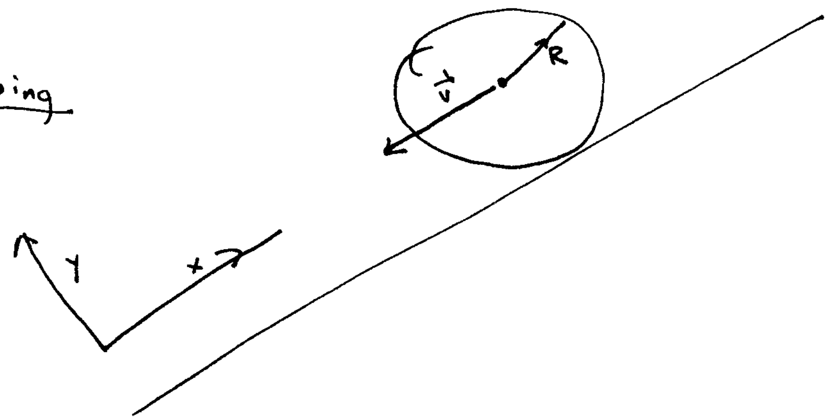
So when the system is the universe, the interaction between the earth and ball is expressed as a potential energy.

When the system is the ball energy and momentum are transferred from the environment as work and impulse.

Section 2.3 - Rolling Wheel.

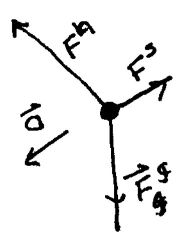
Problem - A wheel of radius R rolls without slipping down an inclined plane. Analyze the motion using wheel as the system.

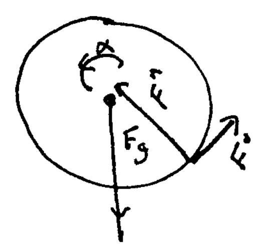
Drawing



Coordinate System - Fixed to earth with x axis along incline.

Free Body **Extended Free Body**





Condition of Rolling - Point on outside of disk must move same distance as center of disk

$$\Delta x_{cm} = \Delta S$$

$$V_{cm} = \Delta \dot{x}_{cm} = \Delta \dot{S} = R\omega$$

<u>Analysis</u> -

$$\vec{P}_t(t) = \int (\Sigma F)\, dt$$

$$K_{sys} = \frac{1}{2} m v_{cm}^2 + \frac{1}{2} I \omega^2$$

$$= \int \vec{F}_g \cdot d\vec{r} \qquad *$$

$\Rightarrow$ Neither static friction nor the normal force does work because the point of application

$$\vec{L}(t) = \int \underbrace{(\vec{r} \times \vec{F})\; dt}_{\text{only static friction provides torque}}$$

<u>Description</u> - The disk will roll down the plane, its angular momentum increasing because of the torque of the force of static friction.

Section 2.4 - Collisions

A collision is any interaction where the interaction takes place over a limited range in space and time. A collision is usually analyzed with momentum and energy.

Dfn Inertia - In any collision

$$\frac{m_1}{m_2} = \frac{|\Delta \vec{v}_2|}{|\Delta \vec{v}_1|}$$

Relative Velocity $\vec{v}_{rel} = \vec{v}_1 - \vec{v}_2$

Elastic Collision - Relative velocity is conserved.
$\Longrightarrow$ Kinetic energy conserved.

Inelastic Collision - Relative Speed $|\vec{v}_{rel}|$ decreases.

Explosive Collision - Relative Speed increases.

Totally Inelastic Collision $|\vec{v}_{rel,f}| = 0$

Coefficient of Restitution $e = \dfrac{|v_{rel,f}|}{|v_{rel,i}|}$

how much of ~~f~~ the velocity is restored after collision

Section 2.2 - Properties Center of Mass

Center of Mass $$\vec{r}_{cm} = \frac{\sum_i m_i \vec{r}_i}{\sum_i m_i}$$

Acceleration of Center of Mass - The acceleration of the center of mass is the same acceleration as a point mass with total mass of the object which experiences the net force on the object.

$$\text{Let } M = \sum_i m_i$$

$$\sum_i \vec{F} = M\vec{a}_{cm}$$

Translational Kinetic Energy - The part of the kinetic energy of a system which cannot be converted to another form in a collision because momentum must be ~~conserved~~ conserved. is

$$K_{\cancel{tran}\ cm} = \frac{1}{2} M v_{cm}^2$$

non-reconvertable

$$K = K_{cm} + \underbrace{K_{sys}}$$ convertable

"Kinetic energy Relative to center of mass"

Free Rotation - Freely Rotating objects rotate about their center of mass.

$\Rightarrow$ A uniform field force like gravity acts like it is applied at the CM and therefore exerts not torque. ~~and on obje~~

$\Rightarrow$ An object rotating in a uniform gravitational field rotates about CM.