

PROBLEMS

4.1 Find the force for each of the following potential energy functions:

- (a) $V = cxyz + C$
- (b) $V = \alpha x^2 + \beta y^2 + \gamma z^2 + C$
- (c) $V = ce^{-\alpha x + \beta y + \gamma z}$
- (d) $V = cr^n$ in spherical coordinates

4.2 By finding the curl, determine which of the following forces are conservative:

- (a) $\mathbf{F} = i\mathbf{x} + j\mathbf{y} + k\mathbf{z}$
- (b) $\mathbf{F} = i\mathbf{y} - j\mathbf{x} + k\mathbf{z}^2$
- (c) $\mathbf{F} = i\mathbf{y} + j\mathbf{x} + k\mathbf{z}^3$
- (d) $\mathbf{F} = -kr^{-n}\mathbf{e}_r$ in spherical coordinates

4.3 Find the value of the constant c such that each of the following forces is conservative:

- (a) $\mathbf{F} = ixy + jcx^2 + kz^3$
- (b) $\mathbf{F} = i(z/y) + cj(xz/y^2) + k(x/y)$

4.4 A particle of mass m moving in three dimensions under the potential energy function $V(x, y, z) = \alpha x + \beta y^2 + \gamma z^3$ has speed v_0 when it passes through the origin.

- (a) What will its speed be if and when it passes through the point $(1, 1, 1)$?
- (b) If the point $(1, 1, 1)$ is a turning point in the motion ($v = 0$), what is v_0 ?
- (c) What are the component differential equations of motion of the particle?

(Note: It is *not* necessary to solve the differential equations of motion in this problem.)

4.5 Consider the two force functions

- (a) $\mathbf{F} = i\mathbf{x} + j\mathbf{y}$
- (b) $\mathbf{F} = i\mathbf{y} - j\mathbf{x}$

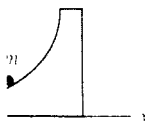
Verify that (a) is conservative and that (b) is nonconservative by showing that the integral $\int \mathbf{F} \cdot d\mathbf{r}$ is independent of the path of integration for (a), but not for (b), by taking two paths in which the starting point is the origin $(0, 0)$, and the endpoint is $(1, 1)$. For one path take the line $x = y$. For the other path take the x -axis out to the point $(1, 0)$ and then the line $x = 1$ up to the point $(1, 1)$.

4.6 Show that the variation of gravity with height can be accounted for approximately by the following potential energy function:

$$V = mgz \left(1 - \frac{z}{r_e} \right)$$

in which r_e is the radius of the Earth. Find the force given by the above potential function. From this find the component differential equations of motion of a projectile under such a force. If the vertical component of the initial velocity is v_{0z} , how high does the projectile go? (Compare with Example 2.3.2.)

4.7 Particles of mud are thrown from the rim of a rolling wheel. If the forward speed of the wheel is v_0 , and the radius of the wheel is b , show that the greatest height above



(4.1)

$$(a) \quad V = cxyz + C$$

$$F = -\nabla V = -c(yz + xz + xy)$$

$$(b) \quad V = \alpha x^2 + \beta y^2 + \gamma z^2 + C$$

$$F = -\nabla V = -(2\alpha x + 2\beta y + 2\gamma z)$$

$$(c) \quad V(x, y, z) = c e^{-(\alpha x + \beta y + \gamma z)}$$

$$F = -\nabla V = -(\alpha + \beta + \gamma) c e^{-(\alpha x + \beta y + \gamma z)}$$

$$(d) \quad V = cr^n$$

$$F = -\nabla V = -\frac{\partial}{\partial r} V \hat{r} = -n c r^{n-1} \hat{e}_r$$

(4.3)

$$(a) \quad \nabla \times F = 0 = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & cx^2 & z^3 \end{vmatrix}$$

$$= \hat{k}(2cx - x) = 0$$

$$c = \frac{1}{2}$$

(b)

$$\nabla \times F = 0 = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{z}{y} & \frac{cxz}{y^2} & \frac{z}{y} \end{vmatrix}$$

$$= i \left(-\frac{z}{y^2} - \frac{cx}{y^2} \right) - j \left(\frac{1}{y} - \frac{1}{y} \right) + k \left(\frac{cx}{y^2} + \frac{z}{y^2} \right)$$

$$c = -1$$