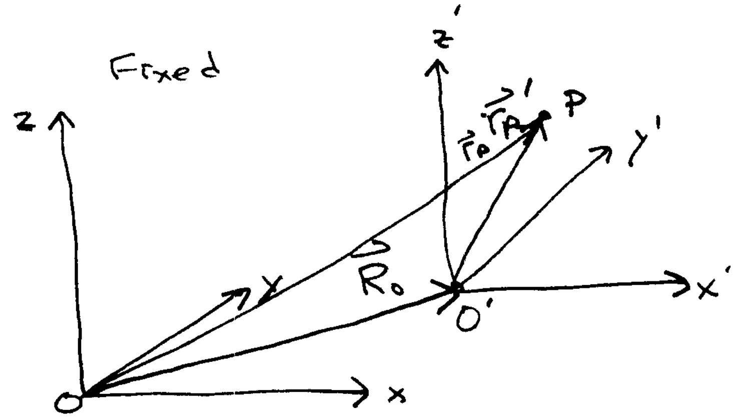
Section 1 - ~~Rotating~~ Accelerating Coordinate Systems

<u>Coordinate System</u> - $Oxyz$

origin and 3 orthogonal unit vectors.

~~Also~~ <u>Moving Coordinate System</u> - $Oxyz \rightarrow O'xyz$

origin changes but unit vectors stay the same.



Let $\vec{R}_o$ be a vector from the origin of the fixed coordinate system to origin of the moving coordinate system.

<u>Describe Location of Point $\vec{P}$</u>

$$\underbrace{\vec{r}_p}_{\text{vector from } O} = \vec{R}_o + \underbrace{\vec{r}_p'}_{\text{vector from } O'}$$

$$\vec{v}(t) = \underbrace{\frac{d\vec{r}_P}{dt}}_{\text{velocity } Oxyz} = \frac{d\vec{R}_O}{dt} + \underbrace{\frac{d\vec{r}_P\,'}{dt}}_{\text{velocity } O'xyz}$$

$$\vec{a}(t) = \frac{d^2\vec{r}_P}{dt^2} = \frac{d^2\vec{R}_O}{dt^2} + \frac{d^2\vec{r}_P\,'}{dt^2}$$

$$\vec{F} = \underbrace{m\vec{a}}_{\text{interial}} = m\frac{d^2\vec{R}_O}{dt^2} + m\frac{d^2\vec{r}_P\,'}{dt^2}$$

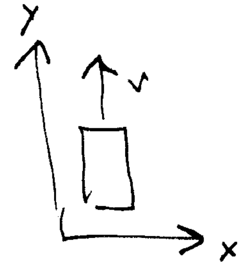
$$= m\vec{A}_O + m\vec{a}\,'$$

If an observer is describing the motion in prime frame, they observe

$$\vec{F}\,' - m\vec{A}_O = m\vec{a}\,'$$

$-mA_O$ is an <u>inertia force</u> a fictious force due to the acceleration of a reference frame.

$$\boxed{\text{Physical forces are the same in an accelerating reference frame}}$$



Example Car accelerates so that it reaches 50 m/s in 10 s.

(a) What is the inertia force felt by a 50 kg rider?

(b) What is the trajectory of a 1 kg ball given an $x_0' = v_0$ initial velocity during acceleration. prime and unprimed frames

Sln Let the road be the fixed frame and the car be the primed frame.

$$\vec{r} = \vec{R}_0 + \vec{r}\,' \qquad \vec{r}\,' = 0$$

$$\vec{v}\,' = 0$$

$$\vec{a} = \vec{A}_0 + \vec{a}\,' \qquad \vec{a}\,' = 0$$

$$\vec{F}_{physical} - m\vec{A}_0 = m\vec{a}_s\,'$$

Find A_0

$$v_y(t) = \overset{=0}{v_0} + A_0 t$$

$$A_0 = \frac{v_y(t)}{t} = \frac{50\ m/s}{10\ s} = 5\ m/s^2$$

$$\vec{A}_0 = 5\ m/s\ \hat{y}$$

Inertial Force $-m\vec{A}_0 = (-50\ kg)(5\ m/s^2)\hat{y} = -250N\hat{y}$

(b)

In moving frame, $\vec{v}_0' = + v_0 \hat{x}$

$$\vec{F}_{phsical} = -mg\,\hat{z}$$

Note: $5\,m/s^2$ is about $g/2$

$$\vec{F}_{physical} - m\vec{A}_0 = m\vec{a}'$$

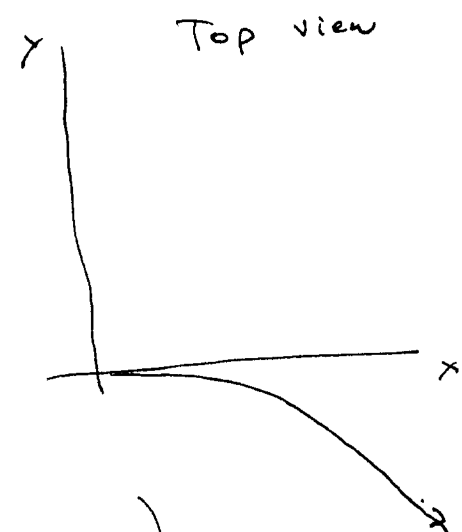
$$-mg\,\hat{z} - mA_0\,\hat{y} = m a'$$

Component EOM

$$m\ddot{x}' = 0 \qquad \dot{x}'(0) = v_0$$

$$m\ddot{y}' = -mA_0 \qquad \dot{y}'(0) = 0$$

$$m\ddot{z}' = -mg \qquad \dot{z}'(0) = 0$$



Sln

$$x'(t) = v_0 t$$

$$y'(t) = -\tfrac{1}{2} A_0 t^2$$

$$z'(t) = -\tfrac{1}{2} g t^2$$

Fixed Frame

$$\vec{v}_0 = (v_{0x}, v_{0y}, 0)$$

$$x(t) = v_{0x} t, \quad y(t) = v_{0y} t$$

$$\vec{F} = -mg\,\hat{z}$$

$$z(t) = -\tfrac{1}{2} g t^2$$

$\Longrightarrow$ Physical (non-inertial) ~~forces~~ are independant of ref~~erance~~ence frame.

Section 2 - Rotating Reference Frames

Let $O = O'$ but let the coordinate axes rotate. Let un-primed system be fixed. The primed systems unit vectors are a function of time

$$\hat{i}'(t),\ \hat{j}'(t),\ \hat{k}'(t)$$

Vectors are Vectors - $\vec{r}$ is the same regardless of coordinate system, but $\vec{r}$ will have different components in different coordinate systems

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k} = x'\hat{i}' + y'\hat{j}' + z'\hat{k}'$$

To find transformation

$$x' = x(\hat{i}\cdot\hat{i}') + y(\hat{j}\cdot\hat{i}') + z(\hat{k}\cdot\hat{i}')$$

$$y' =$$

$$z' =$$

We want $\vec{v}'$ and $\vec{a}'$, but

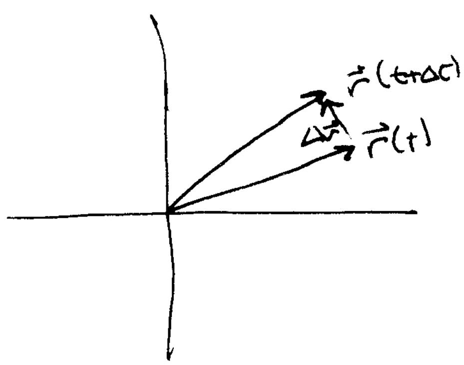
$$\left(\frac{d\vec{r}}{dt}\right)_{fixed} \neq \left(\frac{d\vec{r}}{dt}\right)_{rot}$$

Why?

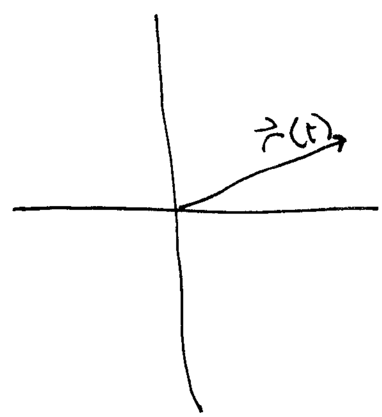
$$\left(\frac{\vec{r}(t+\Delta t)-\vec{r}(t)}{\Delta t}\right)_{fixed} \neq \left(\frac{\vec{r}(t+\Delta t)-\vec{r}(t)}{\Delta t}\right)_{rot}$$

because the coordinate system rotates during the interval Δt.

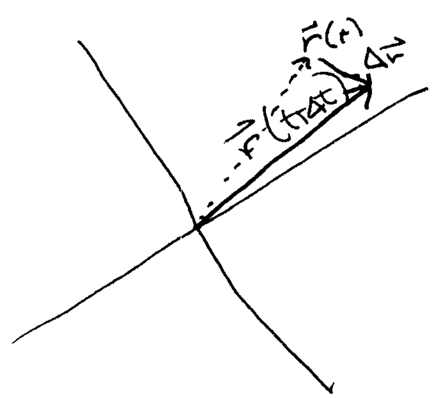
Fixed



Rotating Time t



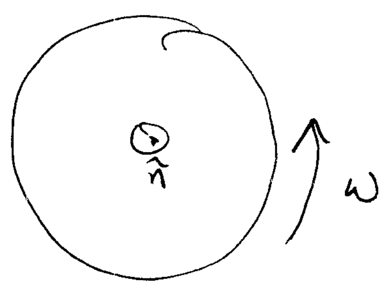
Rotating Time $t+\Delta t$



Rotation - Any rotation may be described as a sequence of infinitesimal rotations about a changing axis of rotation.

Fixed Axis of Rotation - If you curl your fingers in the direction of rotation, your thumb points toward the axis of rotation ($\hat{n}$).

$$\vec{\omega} = \omega \hat{n}$$



For any vector $\vec{A}$

$$\left(\frac{d\vec{A}}{dt}\right)_{\text{fixed}} = \left(\frac{d\vec{A}}{dt}\right)_{\text{rotating}} + \vec{\omega} \times \vec{A}$$

$\Longrightarrow$ That's too useful to be true, let's check it out.

Select a vector that rotates with angular velocity Ω in the fixed frame

$$\vec{r}(t) = R\cos\Omega t\,\hat{\imath} + R\sin\Omega t\,\hat{\jmath}$$

$$\left(\frac{d\vec{r}}{dt}\right)_{fixed} = -R\Omega\sin\Omega t\,\hat{\imath} + R\Omega\cos\Omega t\,\hat{\jmath} = \vec{v}$$

$$= R\Omega\,\hat{e}_\theta$$

$$\left(\frac{d\vec{r}}{dt}\right)_{rot} = \left(\frac{d\vec{r}}{dt}\right)_{fixed} - \vec{\omega}\times\vec{r}$$

Look at a rotating frame $\vec{\omega} = \omega\hat{k}$ (Ask to describe)

$$\vec{\omega}\times\vec{r} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & k \\ 0 & 0 & \omega \\ R\cos\Omega t & R\sin\Omega t & 0 \end{vmatrix}$$

$$= -R\omega\sin\Omega t\,\hat{\imath} + R\omega\cos\Omega t\,\hat{\jmath}$$

$$\left(\frac{d\vec{r}}{dt}\right)_{rot} = \left(\frac{d\vec{r}}{dt}\right)_{fixed} - \vec{\omega}\times\vec{r}$$

$$= \{-R(\Omega-\omega)\sin\Omega t\,\hat{\imath} + R(\Omega-\omega)\cos\Omega t\,\hat{\jmath}$$

The velocity vector in the rotating frame still rotates at Ω, but it has magnitude $R(\Omega-\omega)$, the angular velocity you would see in rotating frame.

Now, get $\left(\frac{d\vec{r}}{dt}\right)_{rot}$ with respect to rotating basis

For any vector,

$$\begin{pmatrix} A_x' \\ A_y' \\ A_z' \end{pmatrix} = \underbrace{\begin{pmatrix} \cos\omega t & \sin\omega t & 0 \\ -\sin\omega t & \cos\omega t & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{Rotation}} \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} = \begin{matrix} (A_x\cos\omega t + A_y\sin\omega t)\hat{i} \\ + \\ (-A_x\sin\omega t + A_y\cos\omega t)\hat{j} \\ + \\ A_z\hat{k} \end{matrix}$$

$$\hat{i}' = \cos\omega t\,\hat{i} + \sin\omega t\,\hat{j}$$

$$\hat{j}' = -\sin\omega t\,\hat{i} + \cos\omega t\,\hat{j}$$

$\boxed{\text{chose sign s.t. rotates correct direction}}$

$$\boxed{\begin{aligned} \hat{i}\cdot\hat{i}' &= \cos\omega t \\ \hat{i}\cdot\hat{j}' &= -\sin\omega t \\ \hat{j}\cdot\hat{j}' &= \cos\omega t \\ \hat{i}'\cdot\hat{j} &= \sin\omega t \end{aligned}}$$

Transform $\vec{r}$

For any vector $\vec{A}$

$$A_x'\hat{i}' + A_y'\hat{j}' = A_x\hat{i} + A_y\hat{j}$$

$$A_x' = A_x\,\hat{i}\cdot\hat{i}' + A_y\,\hat{i}'\cdot\hat{j} = A_x\cos\omega t + A_y\sin\omega t$$

$$A_y' = A_x\,\hat{i}\cdot\hat{j}' + A_y\,\hat{j}\cdot\hat{j}' = -A_x\sin\omega t + A_y\cos\omega t$$

$$\cancel{(v_x')_{rot} = (v_x')_{fxd}\cos\omega t + (v_y')_{fxd}}$$

With respect to rotating coordinate system,

$$v_x' = -R(\Omega-\omega)\cos\omega t\sin\Omega t + R(\Omega-\omega)\cos\Omega t\sin\omega t$$

$$= -R(\Omega-\omega)\sin(\Omega-\omega)t \qquad *\text{ use trig ident.}$$

$$v_y' = R(\Omega-\omega)\sin\Omega t\sin\omega t + R(\Omega-\omega)\cos\Omega t\cos\omega t$$

$$= R(\Omega-\omega)\cos(\Omega-\omega)t$$

The vector rotates at a slower rate in rotating coordinate system

Lecture 2/21/2003

Notes

(1) If a problem ask for a force, but doesn't give a mass report

m, numerical a

(2) Typo in problem 5,

$$\vec{r} = \left(v_{0x} t,\ 0,\ v_{0z} t - \tfrac{1}{2} g t^2\right)$$

Section 1 - Rotating Coordinate Systems

For any vector,

$$\left(\frac{d\vec{A}}{dt}\right)_{fixed} = \left(\frac{d\vec{A}}{dt}\right)_{rot} + \vec{\omega}\times\vec{A}$$

For the vector

$$\left(\frac{d\vec{r}}{dt}\right)_{fixed} = \left(\frac{d\vec{r}}{dt}\right)_{rot} + \vec{\omega}\times\vec{r}$$

$$\vec{v} = \vec{v}\,' + \vec{\omega}\times\vec{r}\,'$$

Note $\vec{r} \neq \vec{r}\,'$ if system translating

Same trick with $\vec{v}$

$$\left(\frac{d\vec{v}}{dt}\right)_{fixed} = \left(\frac{d\vec{v}}{dt}\right)_{rot} + \vec{\omega}\times\vec{v}$$

$$= \left(\frac{d}{dt}\left(\vec{v}\,' + \vec{\omega}\times\vec{r}\,'\right)\right)_{rot} + \vec{\omega}\times\left(\vec{v}\,' + \vec{\omega}\times\vec{r}\,'\right)$$

$$= \left(\frac{d\vec{v}\,'}{dt}\right)_{rot} + \left(\frac{d\vec{\omega}}{dt}\times\vec{r}\,'\right)_{rot} + \left(\vec{\omega}\times\frac{d\vec{r}\,'}{dt}\right)_{rot} + \vec{\omega}\times\vec{v}\,' + \vec{\omega}\times(\vec{\omega}\times\vec{r}\,')$$

$$\left(\frac{d\vec{v}}{dt}\right)_{fixed} = \left(\frac{d\vec{v}'}{dt}\right)_{rot} + \dot{\vec{\omega}} \times \vec{r}' + 2\vec{\omega} \times \vec{v}' + \vec{\omega} \times (\vec{\omega} \times \vec{r}')$$

$$\vec{a} = \underline{\vec{a}' + \dot{\vec{\omega}} \times \vec{r}' + 2\vec{\omega} \times \vec{v}' + \vec{\omega} \times (\vec{\omega} \times \vec{r}')} + \vec{A}_0$$

$$\vec{F}_{physical} = m\vec{a} = m(\quad)$$

or

$$\vec{F}_{physical} + \vec{F}_{coriolis} + \vec{F}_{transverse} + \vec{F}_{centrifual} - \underline{m\vec{A}_0} = m\vec{a}'$$

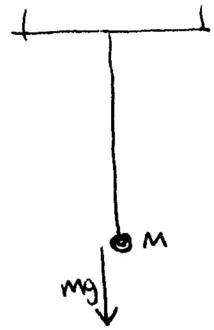
$$\vec{F}_{coriolis} = -2m\vec{\omega} \times \vec{v}'$$

$$\vec{F}_{transverse} = -m\dot{\vec{\omega}} \times \vec{r}'$$

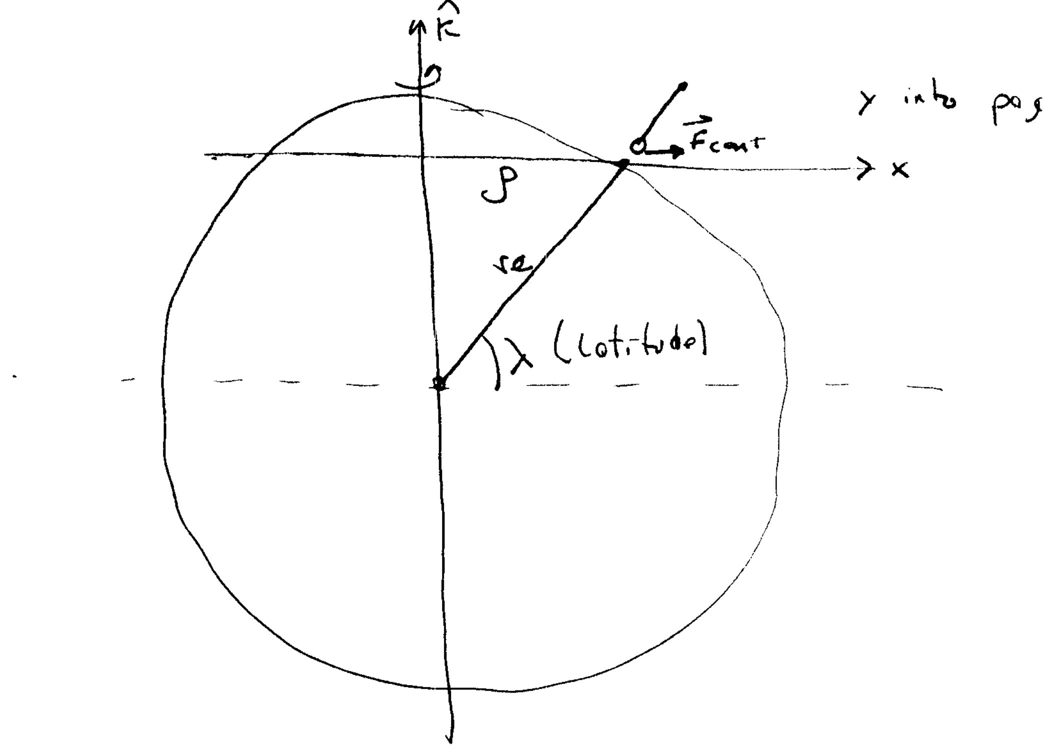
$$\vec{F}_{centarfugal} = -m\vec{\omega} \times (\vec{\omega} \times \vec{r}')$$

Section 2 - Plumb bob

Near the earth's surface, downward can be found by the direction of a mass suspended from a string



But the earth is a rotating coordinate system,



$\rho = r_e \cos \lambda$ $\qquad \vec{\omega} = \omega \vec{k}$ $\qquad \omega = \dfrac{2\pi}{\text{day}}$

Inertial forces $-m\vec{A}_0 = 0$

$$\vec{F}_{cor} = 0 \qquad (\vec{v}' = 0)$$

$$\vec{F}_{trans} = 0 \qquad \dot{\vec{\omega}} = 0$$

$$\vec{F}_{contrifugal} = -m\,\vec{\omega} \times (\vec{\omega} \times \vec{r}\,')$$

$$\vec{r}\,' = \rho\,\hat{\imath}$$

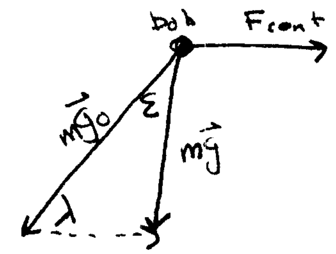
$$\vec{\omega} \times \vec{r}\,' = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & 0 & \omega \\ \rho & 0 & 0 \end{vmatrix} = +\omega\rho\,\hat{\jmath}$$

$$\vec{\omega} \times (\vec{\omega} \times \vec{r}\,') = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & 0 & \omega \\ 0 & \omega\rho & 0 \end{vmatrix} = -\omega^2\rho\,\hat{\imath}$$

$$\vec{F}_{cent} = m\omega^2\rho\,\hat{\imath}$$

$m\vec{g}_o \equiv$ Physical force of gravity

$m\vec{g} \equiv$ Apparent Force of gravity



$$m\vec{g} = m\vec{g}_o + \vec{F}_{cent}$$

$\varepsilon \equiv$ Angle of Deflection from $\vec{g}_o$

Law of Sines

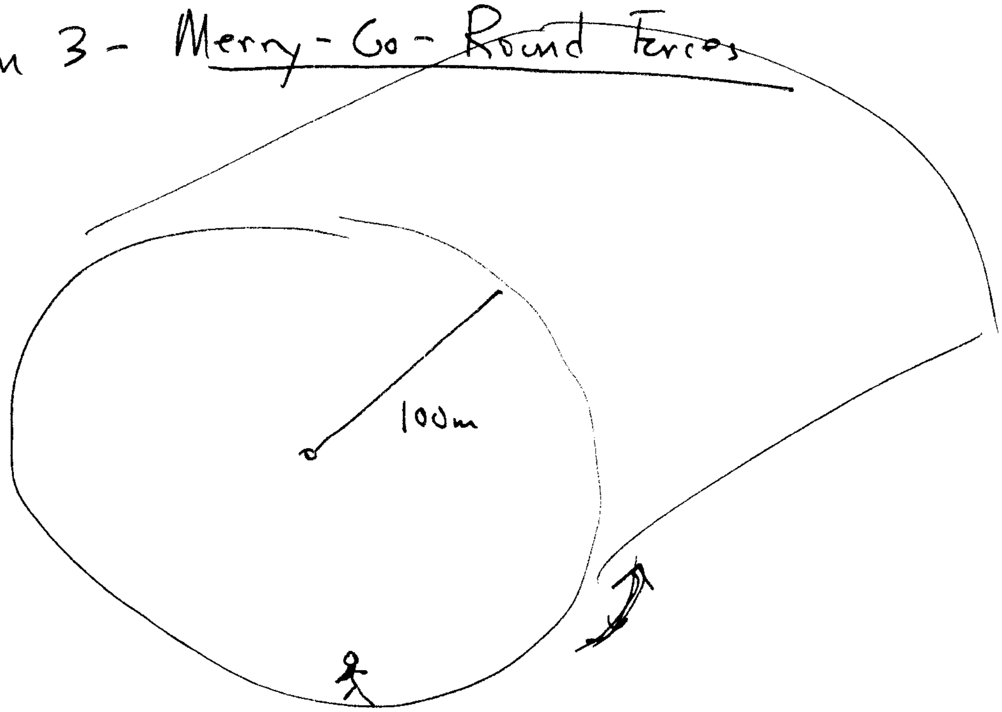
$$\frac{mg}{\sin\lambda} = \frac{m\omega^2 \rho}{\sin\varepsilon}$$

$$\boxed{\sin\varepsilon = \frac{\omega^2 \rho}{g}\sin\lambda = \frac{\omega^2 r_e}{g}\cos\lambda\sin\lambda = \frac{\omega^2 r_e}{2g}\sin 2\lambda}$$

Section 3 - Merry-Go-Round Forces

space

$g=0$



How fast to spin to for centrifugal force to balance gravity?

$$mg = m\frac{v^2}{r} = mr\omega^2 \qquad \omega = \frac{s}{R}$$

$$\omega = \sqrt{\frac{g}{r}} = \sqrt{\frac{9.81\,m/s^2}{100\,m}} = 0.313\,s^{-1}$$

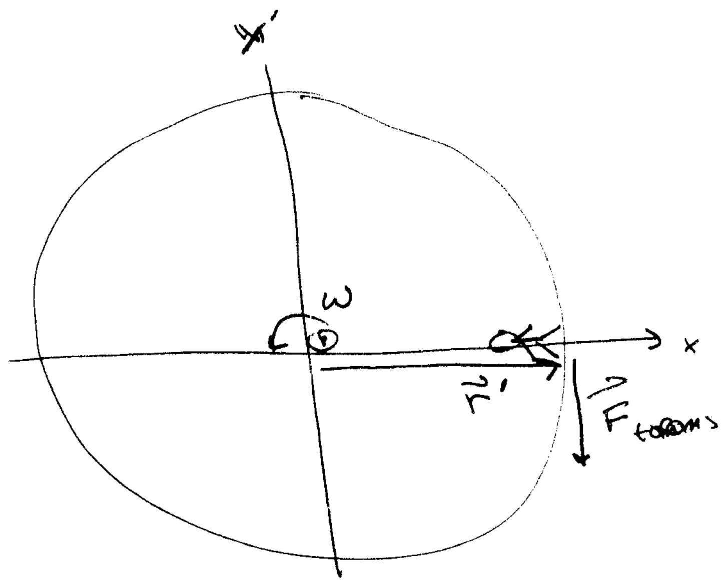
$$\boxed{\text{Period} = \frac{2\pi}{\omega} = 20\,s}$$ wow!

~~How would yo~~ How would you get it to spin? — Set something spinning the other way, conserve angular momentum.

Suppose we don't want a transverse force > g/8 during spin up and spin up is linear, that is

$$\omega(t) = \alpha t \qquad t < t_f$$

$$\vec{F}_{transverse} = -m(\dot{\vec{\omega}} \times \vec{r}\,')$$



$$\vec{r}\,' = r\hat{\imath}$$

$$\dot{\vec{\omega}} = \alpha\hat{k}$$

$$\dot{\vec{\omega}} \times \vec{r}\,' = \alpha r\hat{\jmath}$$

$$\vec{F}_{trans} = -m\alpha r\hat{\jmath} < \frac{mg}{8}\hat{\jmath}$$

$$\alpha = \frac{g}{8r} = 0.011\ s^{-2}$$

Time to spin up $\qquad \omega = 0.313\ s^{-1} = \alpha t_{spin}$

$t_{spin} = \frac{\omega}{\alpha} = 283\ s \approx 5$ minutes

not bad.

<u>Walk down cylinder</u> $v = v_0 \hat{k}$

$$\vec{F}_{cor} = -2\vec{\omega} \times \vec{v}' = 0$$

<u>Walk around cylinder</u> $\vec{v} = v_0 \hat{j}$

$$\vec{F}_{cor} = -2\vec{\omega} \times \vec{v}' = -2\omega v_0 (\hat{k} \times \hat{j}) = 2\omega v_0 \hat{x}$$

<u>Normal Walking</u> $4\,mph = 4 * 0.447 \frac{m}{s} = 1.8\ m/s$

$1\,mph = 0.447\ m/s$

$$\vec{F}_{cor} = 2m(0.313\ s^{-1})(1.8\ m/s)\hat{\imath} = \left(1.13 \frac{m}{s^2}\right) m\, \hat{\imath}$$

$$= (0.1 g)\, m\, \hat{\imath}$$

Heavier

~~At my daughter's (II) running rate, 7.5 mph~~

If you drove around the outside at 20 mph, you would experience $g/2$ addition gravity.

Throw along axis at 50 mph = $-22 m/s\ \hat{x}$

$$\vec{F}_{cor} = -2m\,\vec{\omega}\times\vec{v} = -2m\omega v_0(-\hat{j})$$

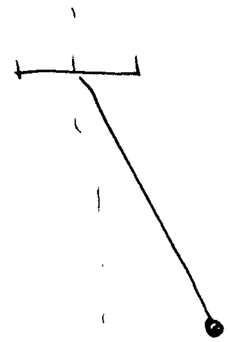
$$\frac{|\vec{F}_{cor}|}{mg} = \frac{2\omega v_0}{g} = 1.4$$

Lecture 4/24/2003

Rotation and Gravitation

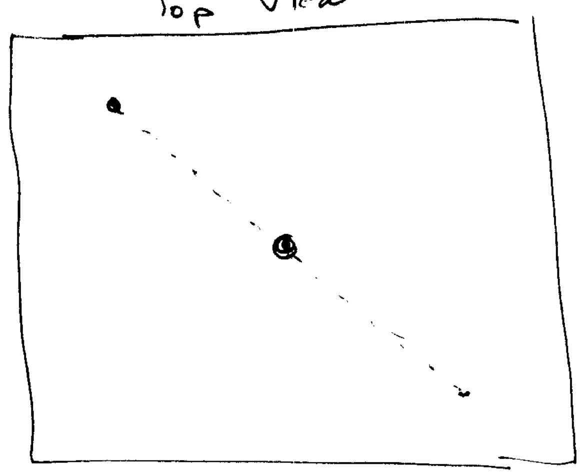
Section 4 - Foucault's Pendulum

Spherical Pendulum

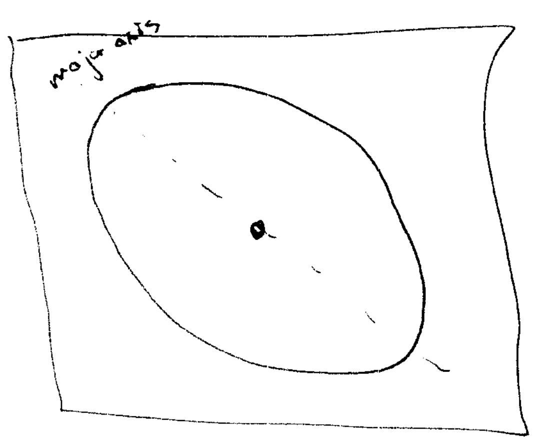


If no $\hat{e}_\theta$ initial velocity, oscillates on a line through center of support

Top View

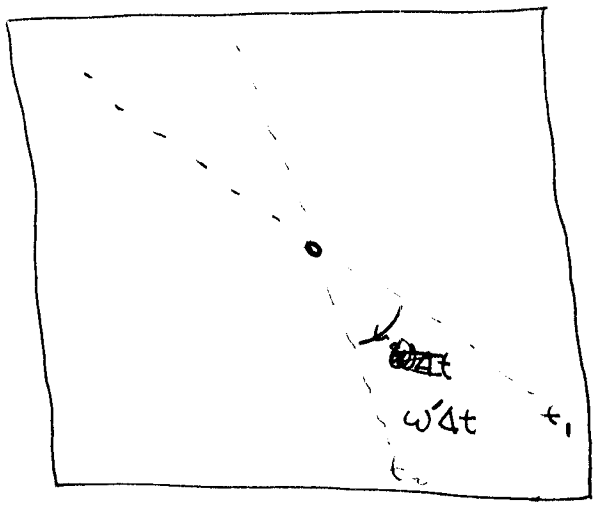


If $\hat{e}_\theta$ initial velocity, travels in ellipse.



Period of Oscillation $- \omega_0 = \sqrt{g/\ell}$

Because of rotation of earth and hence the Coriolis force the major axis precesses (rotates), clockwise in the Northern hemisphere, counterclockwise in the south



Rate of Precession

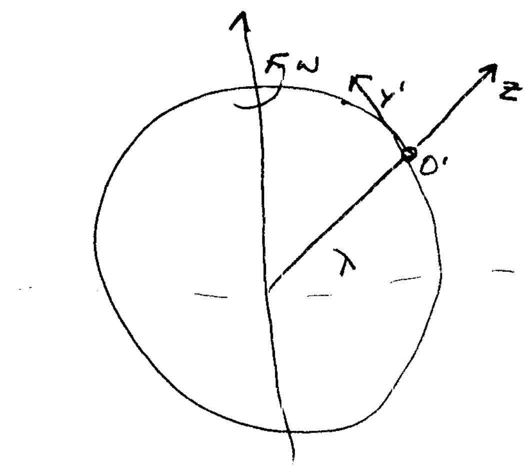
The angular frequency of the precession, the rate the axis rotates is $\omega' = \omega \sin\lambda$, where ω is earth's rotation rate.

Section 5 - Motion Near Rotating Earth

Let $Oxyz$ Fixed (z earths axis)

Let $O'x'y'z'$ be fixed to the earth
with x' (East), y' (North), z' (Up)

$\Rightarrow$ O' is accelerating in circle about earth.



x' into page.

If a projectile (no air resistance) is launched
at $x'(0) = y'(0) = z'(0) = 0$ with an initial
velocity $\dot{x}'_0$, $\dot{y}'_0$, $\dot{z}'_0$ then the trajectory is:

S.4.13

$$x'(t) = \dot{x}'_0 t + \frac{1}{3}\omega g t^3 \cos\lambda - \omega t^2 \left(\dot{z}'_0 \cos\lambda - \dot{y}'_0 \sin\lambda\right)$$

$$y'(t) = \dot{y}'_0 t - \omega \dot{x}'_0 t^2 \sin\lambda$$

$$z'(t) = \dot{z}'_0 t - \frac{1}{2} g t^2 + \omega \dot{x}'_0 t^2 \cos\lambda$$