On substituting the expressions for the primed quantities and their derivatives from the preceding equations into Equations 5.6.3a and b, the following result is obtained, after collecting terms and dropping terms involving ω'^2.

$$\left(\ddot{x} + \frac{g}{l}x\right)\cos\omega't + \left(\ddot{y} + \frac{g}{l}y\right)\sin\omega't = 0 \tag{5.6.5}$$

and an identical equation, except that the sine and cosine are reversed. Clearly, the preceding equation is satisfied if the coefficients of the sine and cosine terms both vanish, namely,

$$\ddot{x} + \frac{g}{l}x = 0 \tag{5.6.6a}$$

$$\ddot{y} + \frac{g}{l}y = 0 \tag{5.6.6b}$$

These are the differential equations of the two-dimensional harmonic oscillator discussed previously in Section 4.4. Thus, the path, projected on the xy plane, is an ellipse with *fixed* orientation in the unprimed system. In the primed system the path is an ellipse that undergoes a steady precession with angular speed $\omega' = \omega \sin \lambda$.

In addition to this type of precession, there is another *natural* precession of the spherical pendulum, which is ordinarily much larger than the rotational precession under discussion. However, if the pendulum is carefully started by drawing it aside with a thread and letting it start from rest by burning the thread, the natural precession is rendered negligibly small.[3]

The rotational precession is clockwise in the Northern Hemisphere and counterclockwise in the Southern. The period is $2\pi/\omega' = 2\pi/(\omega \sin \lambda) = 24/\sin \lambda$ h. Thus, at a latitude of 45°, the period is $(24/0.707)$ h $= 33.94$ h. The result was first demonstrated by the French physicist Jean Foucault in Paris in the year 1851. The Foucault pendulum has come to be a traditional display in major planetariums throughout the world.

PROBLEMS

5.1 A 120-lb person stands on a bathroom spring scale while riding in an elevator. If the elevator has (a) upward and (b) downward acceleration of $g/4$, what is the weight indicated on the scale in each case?

5.2 An ultracentrifuge has a rotational speed of 500 rps. (a) Find the centrifugal force on a 1-μg particle in the sample chamber if the particle is 5 cm from the rotational axis. (b) Express the result as the ratio of the centrifugal force to the weight of the particle.

5.3 A plumb line is held steady while being carried along in a moving train. If the mass of the plumb bob is m, find the tension in the cord and the deflection from the local vertical if the train is accelerating forward with constant acceleration $g/10$. (Ignore any effects of Earth's rotation.)

[3] The natural precession will be discussed briefly in Chapter 10.

5.4 If, in Problem 5.3, the plumb line is not held steady but oscillates as a simple pendulum, find the period of oscillation for small amplitude.

5.5 A hauling truck is traveling on a level road. The driver suddenly applies the brakes, causing the truck to decelerate by an amount $g/2$. This causes a box in the rear of the truck to slide forward. If the coefficient of sliding friction between the box and the truckbed is 1/3, find the acceleration of the box relative to **(a)** the truck and **(b)** the road.

5.6 The position of a particle in a fixed inertial frame of reference is given by the vector

$$\mathbf{r} = \mathbf{i}(x_0 + R\cos\Omega t) + \mathbf{j}R\sin\Omega t$$

where x_0, R, and Ω are constants.

(a) Show that the particle moves in a circle with constant speed.

(b) Calculate the x'- and y'-components of the velocity of the particle relative to a frame of reference rotating with an angular velocity $\boldsymbol{\omega} = \mathbf{k}\omega$.

(c) Letting the fixed and rotating frames of reference coincide at times $t = 0$ and letting $u' = x' + iy'$, find $u'(t)$, assuming that $\Omega \neq -\omega$. (*Note:* $i = \sqrt{-1}$.)

5.7 A new asteroid is discovered in a circular orbit of radius $4^{1/3}$ AU about the Sun.[4] Its period of revolution is precisely 2 years. Assume that at $t = 0$ it is at its distance of closest approach to the Earth.

(a) Find its coordinates $[x(t), y(t)]$ in a frame of reference fixed to Earth but whose axes remain fixed in orientation relative to the distant stars. Assume that the x-axis points toward the asteroid at $t = 0$.

(b) Calculate the velocity of the asteroid, relative to Earth, at $t = 0$.

(c) Find the x- and y-components of the acceleration of the asteroid in this frame of reference. Integrate them twice to show that the resulting positions as a function of time agree with part (a).

(d) Plot the trajectory of the asteroid as seen from this nonrotating frame of reference attached to Earth for its 2-year orbital period. (*Hint: Use Mathematica's* ParametricPlot *graphing tool.*)

5.8 A cockroach crawls with constant speed in a circular path of radius b on a phonograph turntable rotating with constant angular speed ω. The circular path is concentric with the center of the turntable. If the mass of the insect is m and the coefficient of static friction with the surface of the turntable is μ_s, how fast, relative to the turntable, can the cockroach crawl before it starts to slip if it goes (a) in the direction of rotation and (b) opposite to the direction of rotation?

5.9 In the problem of the bicycle wheel rounding a curve, Example 5.2.2, what is the acceleration relative to the ground of the point at the very front of the wheel?

5.10 If the bead on the rotating rod of Example 5.3.3 is initially released from rest (relative to the rod) at its midpoint calculate **(a)** the displacement of the bead along the rod as a function of time; **(b)** the time; and **(c)** the velocity (relative to the rod) when the bead leaves the end of the rod.

5.11 On the salt flats at Bonneville, Utah (latitude = 41°N) the British auto racer John Cobb in 1947 became the first man to travel at a speed of 400 mph on land. If he was headed due north at this speed, find the ratio of the magnitude of the Coriolis force on the racing car to the weight of the car. What is the direction of the Coriolis force?

[4] The radius of the Earth's nearly circular orbit is 1 AU, or 1 astronomical unit. Both the Earth and the asteroid revolve counterclockwise in a common plane about the Sun as seen from the north pole star, Polaris.

5.12 A particle moves in a horizontal plane on the surface of the Earth. Show that the magnitude of the horizontal component of the Coriolis force is independent of the direction of the motion of the particle.

5.13 If a pebble were dropped down an elevator shaft of the Empire State Building (h = 1250 ft, latitude = 41°N), find the deflection of the pebble due to Coriolis force.

5.14 In Yankee Stadium, New York, a baseball is driven a distance of 200 ft in a fairly flat trajectory. Is the amount of deflection due to the Coriolis force alone of much importance? (Let the angle of elevation be 15°.)

5.15 Show that the third derivative with respect to time of the position vector (jerk) of a particle moving in a rotating coordinate system in terms of appropriate derivatives in the rotating system is given by

$$\dddot{\mathbf{r}} = \dddot{\mathbf{r}}' + 3\dot{\boldsymbol{\omega}} \times \ddot{\mathbf{r}}' + 3\boldsymbol{\omega} \times \ddot{\mathbf{r}}' + \ddot{\boldsymbol{\omega}} \times \mathbf{r}' + 3\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \dot{\mathbf{r}}')$$
$$+ \dot{\boldsymbol{\omega}} \times (\boldsymbol{\omega} \times \mathbf{r}') + 2\boldsymbol{\omega} \times (\dot{\boldsymbol{\omega}} \times \mathbf{r}') - \omega^2(\boldsymbol{\omega} \times \mathbf{r}')$$

5.16 A bullet is fired straight up with initial speed v_0'. Assuming g is constant and ignoring air resistance, show that the bullet will hit the ground west of the initial point of upward motion by an amount $4\omega v_0'^3 \cancel{\cos} \lambda/3g^2$, where λ is the latitude and ω is Earth's angular velocity.

5.17 If the bullet in Problem 5.16 is fired due east at an elevation angle α from a point on Earth whose latitude is $+\lambda$, show that it will strike the Earth with a lateral deflection given by $4\omega v_0'^3 \sin\lambda \sin^2\alpha \cos\alpha/g^2$.

5.18 A satellite travels around the Earth in a circular orbit of radius R. The angular speed of a satellite varies inversely with its distance from Earth according to $\omega^2 = k/R^3$, where k is a constant. Observers in the satellite see an object moving nearby, also presumably in orbit about Earth. In order to describe its motion, they use a coordinate system fixed to the satellite with x-axis pointing away from Earth and y-axis pointing in the direction in which the satellite is moving. Show that the equations of motion for the nearby object with respect to the observers' frame of reference are given approximately by

$$\ddot{x} - 2\omega\dot{y} - 3\omega^2 x = 0$$
$$\ddot{y} + 2\omega\dot{x} = 0$$

(see Example 2.3.2 for the force of gravity that Earth exerts on an object at a distance r from it and ignore the gravitational effect of the satellite on the object).

5.19 The force on a charged particle in an electric field $\mathbf{E}$ and a magnetic field $\mathbf{B}$ is given by

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

in an inertial system, where q is the charge and $\mathbf{v}$ is the velocity of the particle in the inertial system. Show that the differential equation of motion referred to a rotating coordinate system with angular velocity $\boldsymbol{\omega} = -(q/2m)\mathbf{B}$ is, for small ω,

$$m\ddot{\mathbf{r}}' = q\mathbf{E}$$

that is, the term involving $\mathbf{B}$ is eliminated. This result is known as *Larmor's theorem.*

5.20 Complete the steps leading to Equation 5.6.5 for the differential equation of motion of the Foucault pendulum.

5.21 The latitude of Mexico City is approximately 19°N. What is the period of precession of a Foucault pendulum there?

5.22 Work Example 5.2.2 using a coordinate system that is fixed to the bicycle wheel and rotates with it, as in Example 5.2.1.

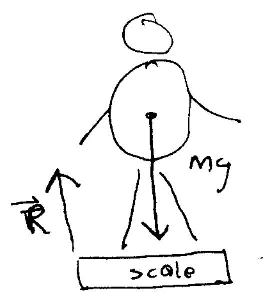
5.1 Choose coordinate system moving with elevator.

(a)

$$\vec{a} = \vec{A}_o + \vec{a}' \qquad \vec{a}' = 0$$

$$\vec{A}_o = \frac{g}{4}\hat{k}$$

$$\vec{F}_{physical} = -mg\hat{k} + \vec{R}$$



$$\vec{F} = m\vec{A}^o + m\vec{a}'$$

$$-mg\hat{k} + \vec{R} = m\vec{A}_o + m\vec{a}' = m\frac{g}{4}\hat{k}$$

$$\vec{R} = \frac{5}{4}mg\hat{k}$$

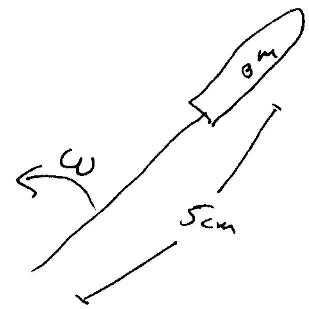
so the person weighs $\frac{5}{4}(120\,lb) = 150\,lb$

(b) Similarly, for downward

$$3/4\,(120\,lb) = 90\,lb.$$

5.2 $f = 500$ rps

$\omega = 1000\,\pi\, s^{-1}$



$$F_{centerfugal} = -F_{centripetal}$$

$$= \frac{m v^2}{r} \qquad \frac{v}{r} = \omega$$

$$= m r \omega^2$$

$$F_{centrifugal} = m r \omega^2 \text{ outward.}$$

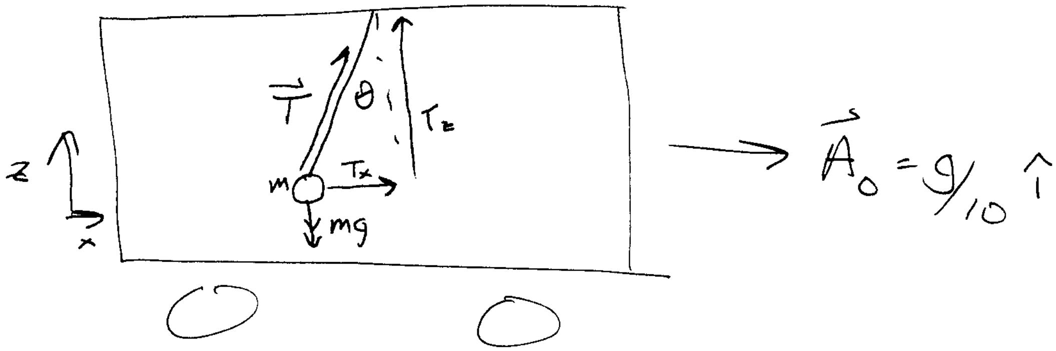
$$1 \mu g = 10^{-9}\,kg$$

$$F_{cent} = (10^{-9} kg)(0.05 m)(1000\,\pi s^{-1})^2$$

$$= 4.9 \times 10^{-4}\,N$$

(b) $$\frac{F_{cent}}{mg} = \frac{4.9 \times 10^{-4} N}{10^{-9} kg \; 9.81\, m/s^2} = 50000$$

5.3



Use train as coordinate system

$$\vec{F}_{physical} = m\vec{A}_0 + m\vec{a}' \qquad \vec{a}' = 0$$

$$\vec{F}_{phys} = -mg\hat{k} + \vec{T}$$

$$\vec{F}_{physical} = -mg\hat{k} + \vec{T} = m\vec{A}_0 = \frac{mg}{10}\hat{\imath}$$

$$\vec{T} = mg\hat{k} + \frac{mg}{10}\hat{\imath}$$

$$T_x = \frac{mg}{10} \qquad T_z = mg$$

$$\tan\theta = \frac{T_x}{T_z} = \frac{1/10\ mg}{mg}$$

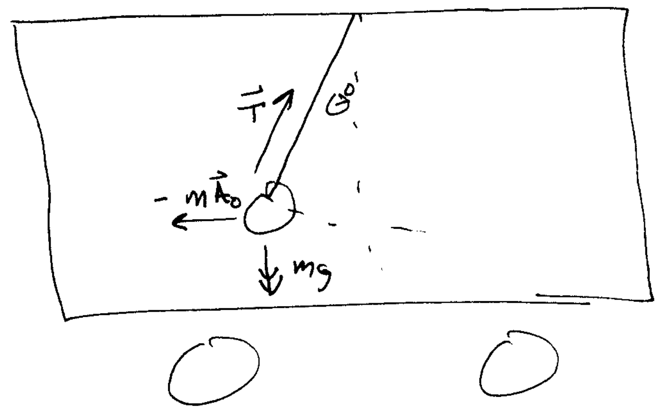
$$= 1/_{10}$$

$$\theta = 5.7^\circ \qquad |\vec{T}| = mg\sqrt{1 + (1/10)^2}$$

$$= 1.005\, mg$$

5.4

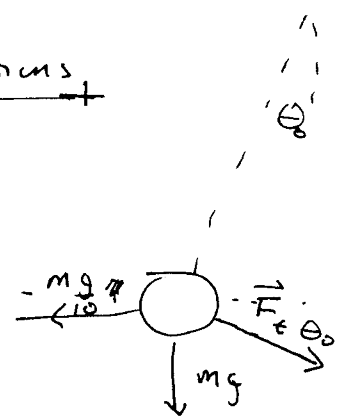
~~Tfile~~



Let $\theta = \theta_0 + \theta'$

$$\vec{F} = -m\vec{A}_0 - mg\hat{k} + \vec{T} = m\vec{a}' = \cancel{ms'} = \cancel{ml\ddot{\theta}}$$

Tangential and Normal Accelerations



$$\vec{F}_t = mg\sin\theta_0 - \frac{mg}{10}\cos\theta_0$$

$$= mg\sin(\theta'+\theta_0) - \frac{mg}{10}\cos(\theta'+\theta_0)$$

$$= m\ddot{s} = ml\ddot{\theta}'$$

$$\sin(\theta' + \theta_0) = \sin\theta' \cos\theta_0 + \cos\theta' \sin\theta_0$$

$$= \theta' \cos\theta_0 + \sin\theta_0$$

$$\cos(\theta' + \theta_0) = \cos\theta' \cos\theta_0 - \sin\theta' \sin\theta_0$$

$$= \cos\theta_0 - \theta' \sin\theta_0$$

$$m l \ddot{\theta}' = mg\left[\theta' \cos\theta_0 + \sin\theta_0\right] - \frac{mg}{10}\left[\cos\theta_0 - \theta' \sin\theta_0\right]$$

$$= \underbrace{\left[mg \sin\theta_0 - \frac{mg}{10} \cos\theta_0\right]}_{\text{Deffects } \theta \text{ may be ignored}} + mg\theta' \cos\theta^0 + \frac{mg}{10} \theta' \sin\theta_0$$

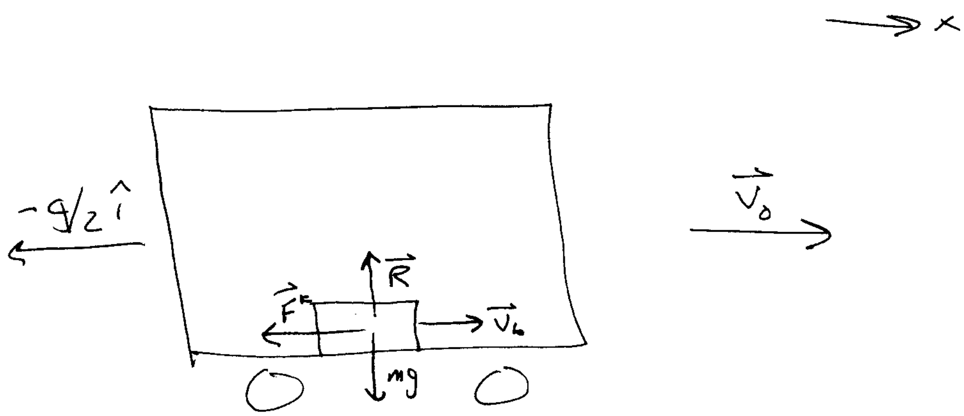
$$m l \ddot{\theta}' = mg\theta'\left[\cos\theta^0 + \frac{\sin\theta_0}{10}\right]$$

$$\omega_0^2 = \frac{g}{l}\left[\cos\theta^0 + \frac{\sin\theta_0}{10}\right]$$

$$= \frac{g}{l}\left[1.005\right]$$

$$\omega_0 = \sqrt{\frac{1.005\, g}{l}}$$

5.5



$$\mu_K = 1/3$$

$$\vec{F} = m\vec{a} = m\vec{A}_0 + m\vec{a}'$$

$$\vec{F} = -\mu^k mg\,\hat{\imath} \underbrace{-mg\,\hat{k} + \vec{R}}_{0} = -\frac{mg}{2}\,\hat{\imath} + m\vec{a}'$$

(a) Truck Frame

$$\vec{a}' = \frac{mg}{2} - \mu^k mg = \frac{mg}{6}$$

(b) Rest Frame

$$\vec{a} = \vec{A}_0 + \vec{a}' = \left(-\frac{g}{2} + \frac{g}{6}\right)\hat{\imath} = -\frac{g}{3}\,\hat{\imath}$$

or the only net force is friction.

5-6 $\vec{r}(t) = x_0 \hat{i} + R\cos\Omega t\, \hat{i} + R\sin\Omega t\, \hat{j}$

$\vec{v}(t) = -R\Omega \sin\Omega t\, \hat{i} + R\Omega\cos\Omega t\, \hat{j}$

(a) $\vec{v}\cdot\vec{v} = v^2 = R^2\Omega^2(\sin^2\Omega t + \cos^2\Omega t) = R^2\Omega^2$

$v = R\Omega$ circular motion

(b) $$\left(\frac{d\vec{r}}{dt}\right)_{fixed} = \left(\frac{dr}{dt}\right)_{rot} + \vec{\omega}\times\vec{r}$$

$\vec{\omega} = \omega\hat{k}$

$$\vec{\omega}\times\vec{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & \omega \\ x_0 + R\cos\Omega t & R\sin\Omega t & 0 \end{vmatrix}$$

$$= -R\omega\sin\Omega t\,\hat{i} + (\omega x_0 + \omega R\cos\Omega t)\hat{j}$$

$$\left(\frac{dr}{dt}\right)_{rot} = \left(\frac{d\vec{r}}{dt}\right)_{fixed} - \vec{\omega}\times\vec{r}$$

$$\left(\frac{dr}{dt}\right)_{rot} = -R\Omega \sin\Omega t\,\hat{i} + R\Omega\cos\Omega t\,\hat{j}$$

$$+ R\omega\sin\Omega t\,\hat{i} - \omega x_0\hat{j} - \omega R\cos\Omega t\,\hat{j}$$

$$= R(\omega-\Omega)\sin\Omega t\,\hat{i} - R(\omega-\Omega)\cos\Omega t\,\hat{j}$$

$$- \omega x_0\hat{j}$$

$$\left(\frac{d\vec{r}}{dt}\right)_{fixed} \cancel{dr} - \vec{\omega}\times\vec{r} = \left(\frac{dr}{dt}\right)_{rot}$$

$$\vec{r} = x'\hat{i} +$$