Homework 2 – Mechanics -Due Tuesday 1/28/2003

Turn Homework in at Dr. Stewart's Mailbox in the Physics Office

Fowles Problems

2.4
2.5
2.9
2.12
2.13
2.14
2.15
2.16

Extra Problems

E1. A system with two stable states can be represented by a potential of the form U(x) = a-bx^2+cx^4

1. Find the location of the minima of this potential.
2. What condition must be met for two minima to exist?
3. When the potential has two minima, find the height of the energy barrier a particle at the bottom of one of the minima must overcome to move to the other stable minima.
4. If a particle of mass m has total energy equal to one tenth the barrier height, compute the trajectory of the particle if the particle is at the location of the minima at t=0. You may expand about the minima.

E2. A motorcyclist jumps a canyon by driving at high speed up a ramp. Analyze the motion using the motorcycle-rider combination as the system. There are three phases of the motion: (1) Two tires on the ramp, (2) One tire on the ramp, and finally (3) No tires on the ramp.

Bonus

B2. What song was played at Jefferson Davis' inauguration?

Difference Between Analytic and Numerical Solutions.

$v_i := 1 - e^{-T_i}$ ← Analytic solution for linear retarding force

$u_i := \dfrac{(e^{2 \cdot T_i} - 1)}{(e^{2 \cdot T_i} + 1)}$ ← Analytic solution for quadratic retarding force

$\Delta uL_i := \dfrac{(v_i - uL_i)}{v_i}$ ← Difference, linear case

$\Delta uQ_i := \dfrac{(u_i - uQ_i)}{u_i}$ ← Difference, quadratic case

PROBLEMS

2.1 Find the velocity $\dot{x}$ and the position x as functions of the time t for a particle of mass m, which starts from rest at $x = 0$ and $t = 0$, subject to the following force functions:

(a) $F_x = F_0 + ct$

(b) $F_x = F_0 \sin ct$

(c) $F_x = F_0 e^{ct}$

where F_0 and c are positive constants.

2.2 Find the velocity $\dot{x}$ as a function of the displacement x for a particle of mass m, which starts from rest at $x = 0$, subject to the following force functions:

(a) $F_x = F_0 + cx$

(b) $F_x = F_0 e^{-cx}$

(c) $F_x = F_0 \cos cx$

where F_0 and c are positive constants.

2.3 Find the potential energy function $V(x)$ for each of the forces in Problem 2.2.

2.4 A particle of mass m is constrained to lie along a frictionless, horizontal plane subject to a force given by the expression $F(x) = -kx$. It is projected from $x = 0$ to the right along the positive x direction with initial kinetic energy $T_0 = 1/2\, kA^2$. k and A are positive constants. Find **(a)** the potential energy function $V(x)$ for this force; **(b)** the kinetic energy, and **(c)** the total energy of the particle as a function of its position. **(d)** Find the turning points of the motion. **(e)** Sketch the potential, kinetic, and total energy functions. (*Optional*: Use *Mathcad* or *Mathematica* to plot these functions. Set k and A each equal to 1.)

2.5 As in the problem above, the particle is projected to the right with initial kinetic energy T_0 but subject to a force $F(x) = -kx + kx^3/A^2$, where k and A are positive constants. Find **(a)** the potential energy function $V(x)$ for this force; **(b)** the kinetic energy, and **(c)** the total energy of the particle as a function of its position. **(d)** Find the turning points of the motion and the condition the total energy of the particle must satisfy if its motion is to exhibit turning points. **(e)** Sketch the potential, kinetic, and total energy functions. (*Optional*: Use *Mathcad* or *Mathematica* to plot these functions. Set k and A each equal to 1.)

2.6 A particle of mass m moves along a frictionless, horizontal plane with a speed given by $v(x) = \alpha/x$, where x is its distance from the origin and α is a positive constant. Find the force $F(x)$ to which the particle is subject.

2.7 A block of mass M has a string of mass m attached to it. A force **F** is applied to the string, and it pulls the block up a frictionless plane that is inclined at an angle θ to the horizontal. Find the force that the string exerts on the block.

2.8 Given that the velocity of a particle in rectilinear motion varies with the displacement x according to the equation

$$\dot{x} = bx^{-3}$$

where b is a positive constant, find the force acting on the particle as a function of x. (*Hint:* $F = m\ddot{x} = m\dot{x}\, d\dot{x}/dx$.)

2.9 A baseball (radius = .0366 m, mass = .145 kg) is dropped from rest at the top of the Empire State Building (height = 1250 ft). Calculate (**a**) the initial potential energy of the baseball, (**b**) its final kinetic energy, and (**c**) the total energy dissipated by the falling baseball by computing the line integral of the force of air resistance along the baseball's total distance of fall. Compare this last result to the difference between the baseball's initial potential energy and its final kinetic energy. (*Hint: In part (c) make approximations when evaluating the hyperbolic functions obtained in carrying out the line integral.*)

2.10 A block of wood is projected up an inclined plane with initial speed v_0. If the inclination of the plane is 30° and the coefficient of sliding friction $\mu_K = 0.1$, find the total time for the block to return to the point of projection.

2.11 A metal block of mass m slides on a horizontal surface that has been lubricated with a heavy oil so that the block suffers a viscous resistance that varies as the 3/2 power of the speed:

$$F(v) = -cv^{3/2}$$

If the initial speed of the block is v_0 at $x = 0$, show that the block cannot travel farther than $2mv_0^{1/2}/c$.

2.12 A gun is fired straight up. Assuming that the air drag on the bullet varies quadratically with speed, show that the speed varies with height according to the equations

$$v^2 = Ae^{-2kx} - \frac{g}{k} \qquad \text{(upward motion)}$$

$$v^2 = \frac{g}{k} - Be^{2kx} \qquad \text{(downward motion)}$$

in which A and B are constants of integration, g is the acceleration of gravity, and $k = c_2/m$ where c_2 is the drag constant and m is the mass of the bullet. (*Note:* x is measured positive upward, and the gravitational force is assumed to be constant.)

2.13 Use the above result to show that, when the bullet hits the ground on its return, the speed will be equal to the expression

$$\frac{v_0 v_t}{(v_0^2 + v_t^2)^{1/2}}$$

in which v_0 is the initial upward speed and

$$v_t = (mg/c_2)^{1/2} = \text{terminal speed} = (g/k)^{1/2}$$

(This result allows one to find the fraction of the initial kinetic energy lost through air friction.)

2.14 A particle of mass m is released from rest a distance b from a fixed origin of force that attracts the particle according to the inverse square law:

$$F(x) = -kx^{-2}$$

Show that the time required for the particle to reach the origin is

$$\pi\left(\frac{mb^3}{8k}\right)^{1/2}$$

2.15 Show that the terminal speed of a falling spherical object is given by

$$v_t = [(mg/c_2) + (c_1/2c_2)^2]^{1/2} - (c_1/2c_2)$$

when *both* the linear and the quadratic terms in the drag force are taken into account.

2.16 Use the above result to calculate the terminal speed of a soap bubble of mass 10^{-7} kg and diameter 10^{-2} m. Compare your value with the value obtained by using Equation 2.4.10.

2.17 Given: The force acting on a particle is the product of a function of the distance and a function of the velocity: $F(x, v) = f(x)g(v)$. Show that the differential equation of motion can be solved by integration. If the force is a product of a function of distance and a function of time, can the equation of motion be solved by simple integration? Can it be solved if the force is a product of a function of time and a function of velocity?

2.18 The force acting on a particle of mass m is given by

$$F = kvx$$

in which k is a positive constant. The particle passes through the origin with speed v_0 at time $t = 0$. Find x as a function of t.

2.19 A surface-going projectile is launched horizontally on the ocean from a stationary warship, with initial speed v_0. Assume that its propulsion system has failed and it is slowed by a retarding force given by $F(v) = -Ae^{\alpha v}$. **(a)** Find its speed as a function of time, $v(t)$. Find **(b)** the time elapsed and **(c)** the distance traveled when the projectile finally comes to rest. A and α are positive constants.

2.20 Assume that a water droplet falling though a humid atmosphere gathers up mass at a rate that is proportional to its cross-sectional area A. Assume that the droplet starts from rest and that its initial radius R_0 is so small that it suffers no resistive force. Show that **(a)** its radius and **(b)** its speed increase linearly with time.

COMPUTER PROBLEMS

C 2.1 A parachutist of mass 70 kg jumps from a plane at an altitude of 32 km above the surface of the Earth. Unfortunately, the parachute fails to open. (In the following parts, neglect horizontal motion and assume that the initial velocity is zero.)

(a) Calculate the time of fall (accurate to 1 s) until ground impact, given no air resistance and a constant value of g.

(b) Calculate the time of fall (accurate to 1 s) until ground impact, given constant g and a force of air resistance given by

$$F(v) = -c_2 v|v|$$

where c_2 is 0.5 in SI units for a falling man and is constant.

Homework Notes

(9) $$W_{diss} = \int F\,dx = c_2 \int v^2\,dx$$

know.

(12) Ignore downward motion formula, work in a coordinate system where x is positive downward. (Downward motion only)

15 and 16 are easy.

(14) Messy integration, see if you can find it in a table or your calc book.

2.4

Force $F(x) = -kx$ is projected to right with initial kinetic energy $T_0 = \frac{kA^2}{2}$

(a) $$F(x) = -\frac{dU}{dx} = -kx$$

$$U(x) = \frac{1}{2}kx^2 + C$$

Let $U = 0$ at origin.

$$U(x) = \frac{1}{2}kx^2$$

(b) The surface is frictionless, so total energy is conserved

$$E = T + U = T_0 + U(0) = \frac{1}{2}kA^2$$

$$E = \frac{1}{2}kA^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

~~The total energy is constant~~.

$$T(x) = \frac{1}{2}kA^2 - \frac{1}{2}kx^2$$

(c) The total energy is constant.

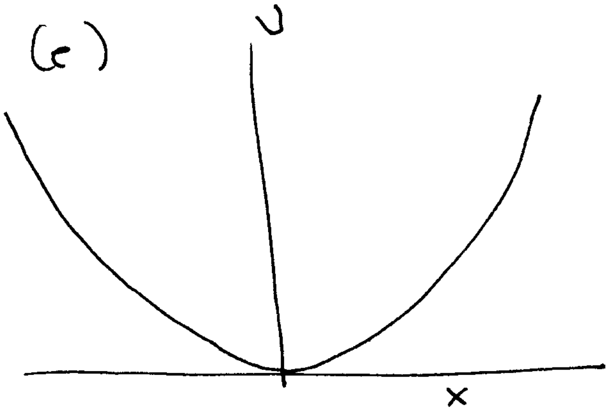
(d) At the turning points all energy is potential

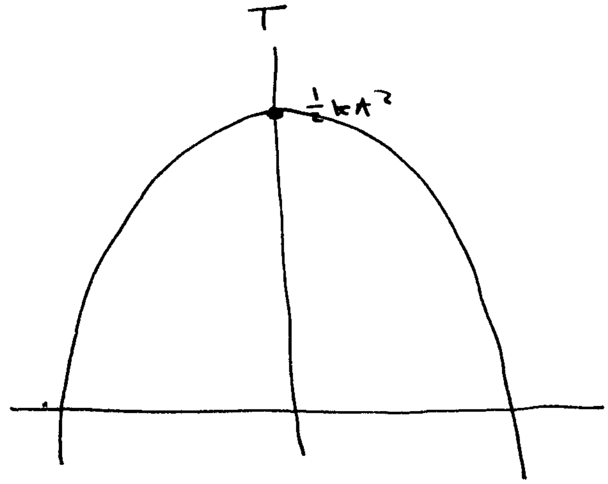
$$E = \frac{1}{2}KA^2 = \frac{1}{2}Kx^2$$

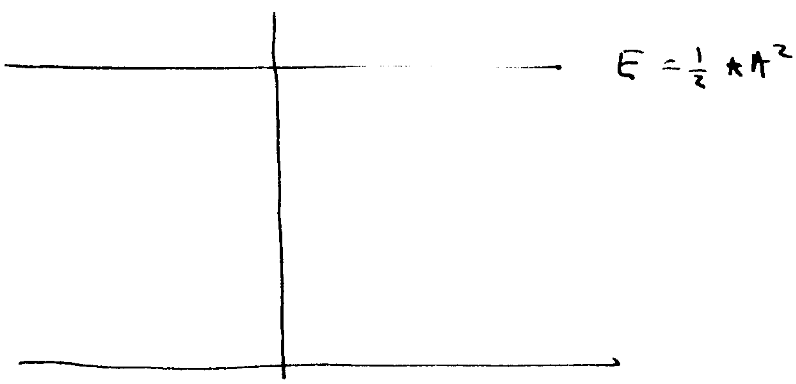
$$x_t = \pm A$$

(2.4)

(e)







2.5

Project to right with initial kinetic energy

$$T_0 = \frac{1}{2} k A^2$$

subject to force $\quad F(x) = -kx + \frac{k x^3}{A^2} = -\frac{dU}{dx}$

(a) $$U(x) = \int \left(+kx - \frac{kx^3}{A^2} \right) dx$$

$$= \frac{1}{2} k x^2 - \frac{1}{4} \frac{k}{A^2} x^4$$

where I have chosen $U(0) = 0$.

(b) The total energy is conserved

$$E = T(x) + U(x) = \cancel{\tfrac{1}{2} k A^2}\ T_0 + U(0)$$

(c) The kinetic energy is

$$T(x) = E - V(x) = \cancel{\tfrac{1}{2} k A^2}\ T_0 - \frac{1}{2} k x^2 + \frac{1}{4} \frac{k}{A^2} x^4$$

(d) Turing points happen when $T(x_t) = 0$

$$\cancel{\tfrac{1}{2} k A^2}\ T_0 - \frac{1}{2} k x^2 + \frac{1}{4} \frac{k}{A^2} x^4 = 0$$

$$\cancel{x^2 = \tfrac{1}{2} k \pm \sqrt{\left(\tfrac{k}{2}\right)^2 - \tfrac{1}{2}}}$$

$$\cancel{x^4 - 2A^2 + 2A^4 = 0}$$

$$\cancel{x^2 = \frac{2A^2 \pm \sqrt{4A^2 - 8A^4}}{2}}$$

$$x^4 - 2A^2x^2 + \frac{4A^2 T_0}{k} = 0$$

<u>Turning Points</u> t

$$x_t^2 = \frac{2A^2 \pm \sqrt{4A^4 - \frac{16A^2T_0}{k}}}{2}$$

$$x_t^2 = A^2 \pm \sqrt{A^4 - \frac{4A^2T_0}{k}}$$

For a turning point to exist, the quantity under the square root must be positive.

$$A^4 - \frac{4A^2T_0}{k} > 0$$

$$A^2 > \frac{4T_0}{k}$$

$$\boxed{\frac{kA^2}{4} > T_0}$$

Work on the potential energy

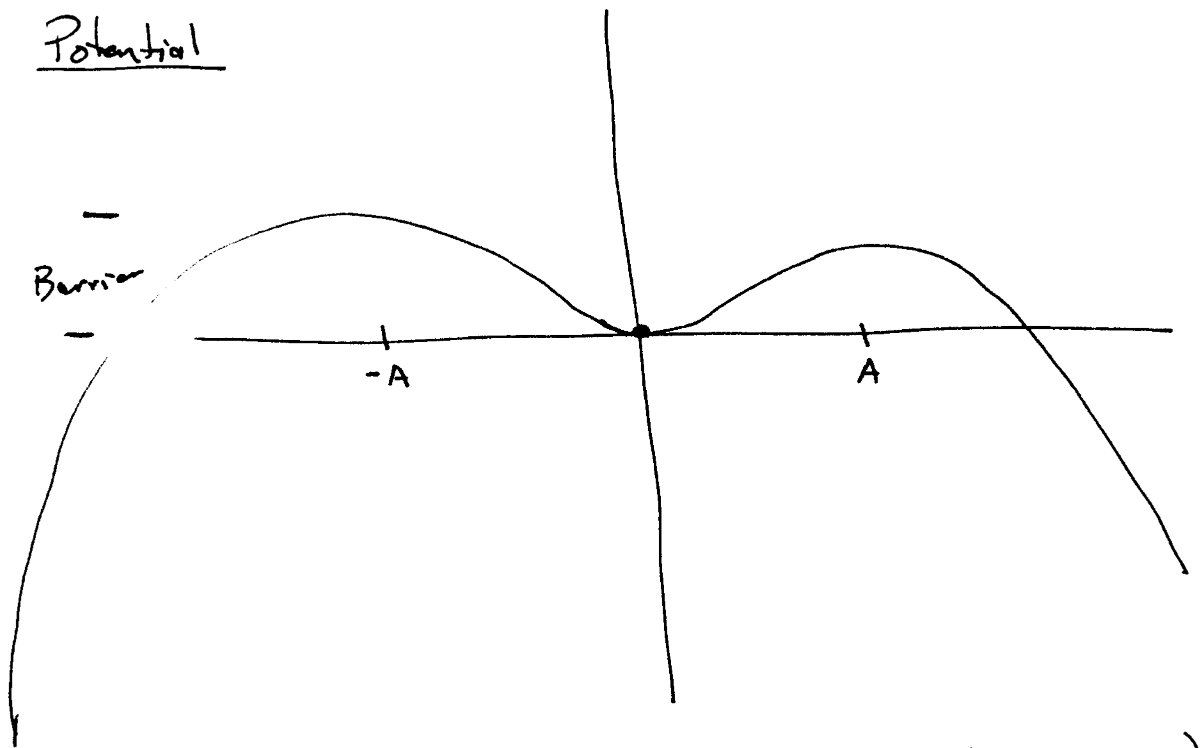
$$U(x) = \frac{1}{2}k\left(x^2 - \frac{1}{2A^2}x^4\right)$$

$$\frac{dU}{dx} = \frac{1}{2}k\left(2x - \frac{2}{A^2}x^3\right) = 0 \quad \text{extrema}$$

$$x = 0 \quad \text{and} \quad \left(1 - \frac{x^2}{A^2}\right)$$

$$x = \pm A$$

Potential



$$\text{Barrier height} = U(A) = \frac{1}{2}k\left(A^2 - \frac{A^2}{2}\right) = T_0$$

$$\frac{kA^2}{4} = T_0$$

(2.9)

Baseball $\quad R = 0.0366\,m \;, \quad m = 0.145\,kg$

$$h_o = 1250\,ft * \frac{0.3048\,m}{1\,ft} = 381\,m$$

(a) Initial Potential Energy

$$U(h_o) = m g h_o =$$

$$(0.145\,kg)(9.81\,\tfrac{m}{s^2})(381\,m) =$$

$$U(h_o) = 542\,J$$

(b) Evaluate which expression for drag is correct -

$$\frac{\text{Quadric Drag}}{\text{Linear Drag}} = 1.4\times10^{3}\,(s/m^2)\,|v|\,D$$

$$= (1.4\times10^{3}\,\tfrac{s}{m^2})(2)(0.0366\,m)$$

$$= 102.5\,\left(\tfrac{s}{m}\right)|v|$$

Since $|v|$ will quickly be larger the $\frac{1}{102.5}\,m/s$ the quadratic term dominates.

$$F_y = -c_2 v^2 + mg = m\frac{dv}{dt} = mv\frac{dv}{dx}$$

Terminal Velocity is reached when the total force on the ball is zero.

$$F_y = 0 = -c_2 v_t^2 + mg$$

$$v_t^2 = \frac{mg}{c_2} \qquad v_t = \sqrt{\frac{mg}{c_2}}$$

1 Compute v_t

$$v_t = \sqrt{\frac{mg}{c_2}} = \sqrt{\frac{mg}{(0.22)(2R)^2}}$$

$$= \sqrt{\frac{(0.145\,kg)(9.81\,m/s^2)}{(0.22)(2 * 0.0366\,m)^2}}$$

$$= 34.7\,m/s$$

Solve for the velocity as a function of time

$$dy = \frac{mv\,dv}{mg - c_2 v^2} = \cancel{\frac{(g/c_2)}{(g/c_2)}}\,\frac{mv\,dv}{mg - c_2 v^2}$$

Let $u = mg - c_2 v^2$

$du = -2c_2 v\, dv$

$$dy = -\frac{m}{2c_2}\frac{du}{u}$$

$$\int_0^y dy = -\frac{m}{2c_2}\int_{u_0}^{u}\frac{du}{u}$$

$$y = -\frac{m}{2c_2}\ln\left(u/u_0\right)$$

$$u = u_0 \exp -\left[\frac{2c_2 y}{m}\right] = u_0 \exp -\left[y/\lambda_t\right]$$

$$\boxed{mg - c_2 v^2 = (mg - c_2 v_0^2)\exp - y/\lambda_t}$$

λ_t is the characteristic distance

$$\lambda_t = \frac{m}{2c_2} = \frac{v_t^2}{2g} \qquad c_2 = \frac{mg}{v_t^2}$$

$$= \frac{(34.7\,m/s)^2}{2(9.81\,m/s^2)} = 61.5\,m$$

So to fall 381m, the ball has traverse

$$\frac{381m}{\lambda_t} = 6 \text{ characteristic distances}$$

so $v \approx v_t$

(b) The final kinetic energy as it strikes the ground. is

$$T_f = \frac{1}{2} m v_t^2 = \frac{1}{2}(0.145\,kg)(34.7\,m/s)^2$$

$$\boxed{T_f = 87.2\,J}$$

So the energy dissipated by air friction is

$$E_{diss} = U_i - T_f = mgh_o - T_f$$

$$= (0.145\,kg)(9.81\,m/s^2)(381\,m) - 87.2\,J$$

$$= 454.7\,J$$

(c) $$E_{diss} = \int_0^h F_{air}\,dy = \int_0^h c_2 v^2\,dy$$

Work on velocity expression let $v_0 = 0$

$$mg\left[1 - \exp\left(-\frac{y}{\lambda_t}\right)\right] = c_2 v^2$$

$$v^2(y) = \frac{mg}{c_2}\left[\quad\right] = v_t^2\left[1 - \exp\left(\frac{-y}{\lambda_t}\right)\right]$$

$$E_{diss} = v_t^2 c_2 \int_0^h \left[1 - \exp -\frac{y}{\lambda_t}\right] dy$$

$$= v_t^2 c_2 h - v_t^2 c_2 \int_0^h \exp -\frac{y}{\lambda_t}\,dy$$

$$\cancel{= v_t^2 c_2 h +}$$

$$u = \frac{x}{\lambda_t} \qquad du = \frac{dy}{\lambda_t}$$

$$\lambda_t \int_0^h e^{-u}\, du = -\lambda_t e^{-u} \Big|_0^{h/\lambda_t}$$

$$= \lambda_t \left[1 - e^{-h/\lambda_t} \right]$$

$$E_{diss} = v_t^2 c_2 h - \lambda_t v_t^2 c_2 \left[1 - e^{-h/\lambda_t} \right]$$

$$= v_t^2 c_2 (h - \lambda_t) + v_t^2 c_2 \lambda_t e^{-h/\lambda_t}$$

$$= mg(h - \lambda_t) + mg\lambda_t e^{-h/\lambda_t}$$

$$= (0.145\,kg)(9.81\,m/s^2)(381\,m - 61.5\,m)$$

$$= 454.5\,J \quad \cancel{+\,0.14}$$

2.12 Bullet Fired Straight Up

$$F_{up} = -mg - c_2 v^2 \qquad F_{down} = -mg + cv^2$$

Up Phase

$$F_{up} = -(mg + c_2 v^2) = mv\frac{dv}{dx}$$

$$\int_0^x dx = \int_{v_0}^{v} -\frac{mv\,dv}{mg + c_2 v^2} = x$$

$$u = mg + c_2 v^2 \qquad du = 2c_2 v\,dv$$

$$x = -\frac{m}{2c_2}\int_{u_0}^{u}\frac{du}{u}$$

$$= -\frac{m}{2c_2}\ln\left(u/u_0\right)$$

Define $k = c_2/m$

$$u(x) = u_0\, e^{-2kx}$$

$$mg + c_2 v^2 = (mg + c_2 v_0^2)\, e^{-2kx}$$

$$v^2 = v_0^2 e^{-2kx} - \frac{mg}{c_2}\left[1 - e^{-2kx}\right]$$

OR

$$v^2 = \left[\frac{mg + c_2 v_0^2}{c_2}\right] e^{-2kx} - \frac{mg}{c_2}$$

<u>Maximum Hieght</u> $\quad v = 0$

$$\frac{mg}{mg + c_2 v_0^2} = e^{-2kx}$$

$$x_{max} = -\frac{1}{2k} \ln\left[\frac{mg}{mg + c_2 v_0^2}\right]$$

<u>Downward Motion</u> – Measure x as positive downward with $x=0$ being the maximum height.

$$F_{down} = +mg - c_2 v^2 = mv\frac{dv}{dx}$$

$$\int_0^x dx = \int_0^v \frac{mv\,dv}{mg - cv^2} \qquad v_0 = 0 \text{ at } t=0$$

$$u = mg - c_2 v^2 \qquad du = -2c_2 v\, dv$$

$$x = -\frac{m}{2c_2}\int_{u_0}^{u} \frac{du}{u}$$

$$-2kx = \ln(u/u_0)$$

$$u = u_0\, e^{-2kx}$$

$$mg - c_2 v^2 = mg\, e^{-2kx}$$

$$v^2 = \frac{mg}{c_2}\left[1 - e^{-2kx}\right]$$

2.13

The speed of the bullet as it hits the ground is

$$v^2(h_f) = v_t^2\left[1 - e^{-2kh_f}\right]$$

where $\cancel{h_f} =$

$$\frac{mg}{mg + c_2 v_0^2} = e^{-2kh_f} = \frac{v_t^2}{v_t^2 + v_0^2}$$

(from previous problem)

$$v^2(h_f) = v_t^2\left[1 - \frac{v_t^2}{v_t^2+v_0^2}\right]$$

$$= \frac{v_t^2 v_0^2}{v_t^2 + v_0^2}$$

$$v(h_f) = \frac{v_t v_0}{\sqrt{v_t^2+v_0^2}}$$

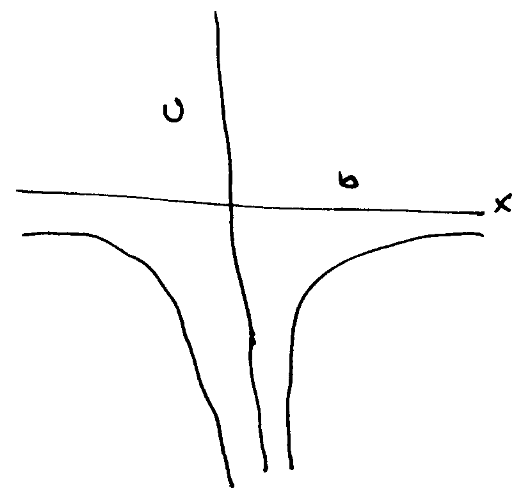
2.14

Particle released at $x_0 = b$ under force

$$F = -\frac{k}{x^2} = -\frac{dU}{dx}$$

$$\frac{dU}{dx} = \frac{k}{x^2}$$

$$U(x) = -\frac{k}{x}$$



Energy is conserved,

$$E_{sys} = -\frac{k}{b} = \frac{1}{2}mv^2 - \frac{k}{x}$$

$$v^2 = \frac{2}{m}\left[\frac{k}{x} - \frac{k}{b}\right]$$

$$v = \left(\frac{2k}{m}\left[\frac{1}{x} - \frac{1}{b}\right]\right)^{1/2} = \frac{dx}{dt}$$

$$x(t) = \sqrt{\frac{2k}{m}}\int_b^x \left(\frac{1}{x} - \frac{1}{b}\right)^{1/2} dx$$

$$= \sqrt{\frac{2k}{mb}}\int\left(\frac{b}{x} - 1\right)^{1/2} dx$$

$$u = \frac{x}{b}$$

$$du = \frac{dx}{b}$$

$$= b\sqrt{\frac{2k}{mb}}\int_1^u \left(\frac{1}{u} - 1\right)^{1/2} du$$

$$\int_0^t dt = t = \int_b^0 \frac{dx}{\sqrt{\frac{2k}{m}}\left(\frac{1}{x} - \frac{1}{b}\right)^{1/2}}$$

Let $\quad u = x/b \qquad du = d\cancel{x}/b$

$$t = b\sqrt{\frac{m}{2k}}\int_1^0 \frac{du}{\left(\frac{1}{ub} - \frac{1}{b}\right)^{1/2}} = \sqrt{\frac{b^3 m}{2k}}\int_1^0 \frac{du}{\left(\frac{1}{u} - 1\right)^{1/2}}$$

$$t = \sqrt{\frac{b^3 m}{2k}}\int_1^0 \frac{\sqrt{u}\, du}{(1-u)^{1/2}}$$

Let $\quad u = \sin^2\theta \qquad du = 2\sin\theta\cos\theta\, d\theta$

$$t = \sqrt{\frac{b^3 m}{2k}}\int_{\pi/2}^0 \frac{\left(\sqrt{\sin^2\theta}\right)(2\sin\theta\cos\theta)\, d\theta}{(1-\sin^2\theta)^{1/2}}$$

$$= \sqrt{\frac{b^3 m}{2k}}\int_{\pi/2}^0 \frac{2\sin^2\theta\cos\theta}{\cos^2\theta}\, d\theta$$

$$= \sqrt{\frac{2b^3 m}{k}}\int_{\pi/2}^0 \sin^2\theta\, d\theta$$

$$= \left(\sqrt{\frac{2b^3 m}{k}}\right)\int_{\pi/2}^0 \left(\frac{1-\cos 2\theta}{2}\right) d\theta$$

$\phi = 2\theta$

$d\phi = 2d\theta$

$$\sqrt{\frac{2b^3 m}{k}}\int_{\pi}^0 \frac{(1-\cos\phi)}{4}\, d\phi$$

$$= \pi\sqrt{\frac{2b^3 m}{\ldots}} \qquad \text{up to a sign}$$

2.15

The force when both drag coefficents are present is

$$F_x = mg - c_1 v - c_2 v^2$$

- $F_x = 0$ at terminal velocity.

$$0 = mg - c_1 v_t - c_2 v_t^2$$

$$v_t^2 + \frac{c_1}{c_2} v_t - \frac{mg}{c_2} = 0$$

$$v_t = \frac{-\frac{c_1}{c_2} \pm \sqrt{\left(\frac{c_1}{c_2}\right)^2 + \frac{4mg}{c_2}}}{2}$$

Take + root,

$$v_t = \frac{-c_1}{2c_2} + \sqrt{\left(\frac{c_1}{2c_2}\right)^2 + \frac{mg}{c_2}}$$

2.16

$m = 10^{-7}\,kg$ $\qquad$ $D = 10^{-2}\,m$

$c_1 = 1.55\times10^{-4}D$ $\qquad$ $c_2 = 0.22\,D^2$

Terminal Velocity (Quadratic)

$$v_{tq} = \sqrt{\frac{mg}{c_2}} = \sqrt{\frac{mg}{0.22\,D^2}}$$

$$= 0.211\ m/s$$

Terminal Velocity (Both)

$$\frac{c_1}{2c_2} = \frac{1.55\times10^{-4}D}{2(0.22)\,D^2}$$

$$= \frac{1.55\times10^{-4}}{2(0.22)D} = 0.035$$

$$v_t = -0.035 + \sqrt{(0.035)^2 + v_{tq}^2}$$

$$= 0.179\ m/s$$ not much of a correction.

E1 $\qquad U = a - bx^2 + cx^4$

① Minima

$$\frac{dU}{dx} = 0 = -2bx + 4cx^3$$

$$\underbrace{x = 0}_{\text{Maxima}} \qquad \underbrace{\boxed{x^{*} = \pm\sqrt{\frac{b}{2c}}}}_{\text{Minima}}$$

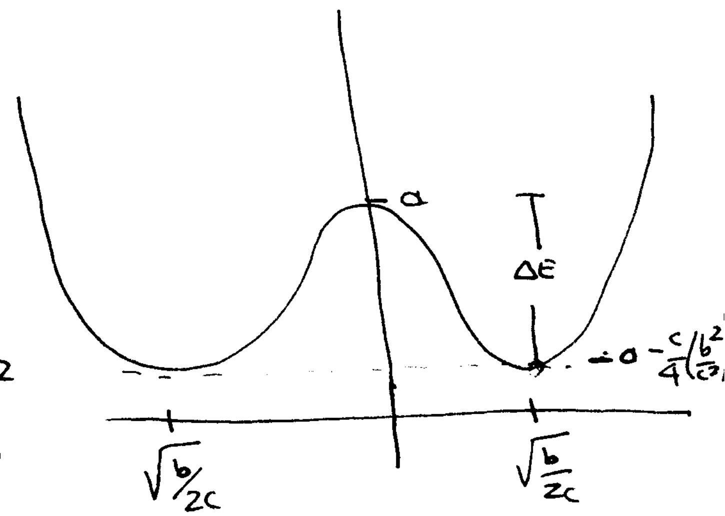
② $bc > 0$ or $b/c > 0$ $\Rightarrow$ $\boxed{c > 0 \quad b > 0}$

If $c < 0$, two maxima.

③

$U(0) = a$

Maximum



$$U\left(\sqrt{\frac{b}{2c}}\right) =$$

$$a - b\left(\frac{b}{2c}\right) + c\left(\frac{b}{2c}\right)^2$$

$$= a - \frac{c}{2}\left(\frac{b^2}{c^2}\right) + \frac{c}{4}\left(\frac{b}{c}\right)^2$$

$$= a - \frac{c}{4}\left(\frac{b^2}{c^2}\right)$$

Barrier Height $\Delta E = a - \left(a - \frac{c}{4}\left(\frac{b^2}{c^2}\right)\right) = \frac{c}{4}\left(\frac{b^2}{c^2}\right)$

(4) Taylor Expand about minima

$$U(x-x_0) = U(x_0) + \underbrace{\left.\frac{dU}{dx}\right|_{x_0}(x-x_0)}_{0} + \frac{1}{2}\left.\frac{d^2U}{dx^2}\right|_{x_0}(x-x_0)^2$$

$$= \left[a - \frac{c}{4}\left(\frac{b^2}{c^2}\right)\right] + \frac{1}{2}\left.\frac{d^2U}{dx^2}\right|_{x_0}(x-x_0)^2$$

$$\left.\frac{d^2U}{dx^2}\right|_{\sqrt{\frac{b}{2c}}} = -2b + 12cx^2\Big|_{\sqrt{b/2c}}$$

$$= -2b + 12c\left(\frac{b}{2c}\right)$$

$$= -2b + 6b = 4b$$

$$U(x-x_0) = \left[a - \frac{c}{4}\left(\frac{b^2}{c^2}\right)\right] + 2b(x-x_0)^2$$

<u>Restoring Force</u> $\quad F = -4b(x-x_0)$

SHO with linear restoring force $-k(x-x_0)$

$$k = 4b$$

$$\boxed{\omega_0^2 = \frac{4b}{m}}$$

Let $x' = x - x_0$

$$x'(t) = A\cos(\omega_0 t + \phi_0) \quad 0 = x'(0) \Rightarrow \phi_0 = \frac{-\pi}{2}$$

$$\dot{x}'(t) = -A\omega_0 \sin(\omega_0 t + \phi_0) \quad v_0 = \dot{x}'(0)$$

$$\frac{1}{2} m v_0^2 = \frac{\Delta E_{barrier}}{10} = T_0$$

$$v_0 = \sqrt{\frac{2T_0}{m}}$$

$$\dot{x}'(0) = -A\omega_0 \sin\left(\frac{-\pi}{2}\right) = A\omega_0 = \sqrt{\frac{2T_0}{m}}$$

$$\boxed{A = \frac{1}{\omega_0}\sqrt{\frac{2T_0}{m}}} \qquad \boxed{\phi_0 = -\frac{\pi}{2}}$$

$$x'(t) = A\cos\left(\omega_0 t - \frac{\pi}{2}\right)$$

E2 Motorcycle travelling up ramp

Possible Assumptions

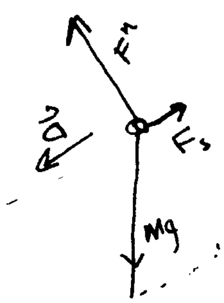
I. Still accelerating

II. Coasting ← I will do this case

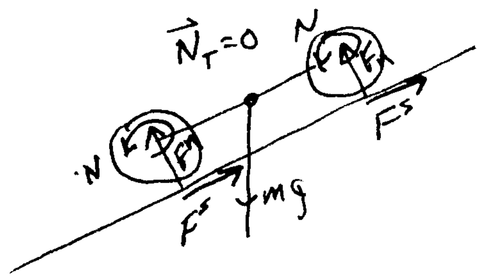
Phase I - Both Wheels on Ramp

Motorcycle is slowing so there must be a torque about the center of the wheel.

Free Body



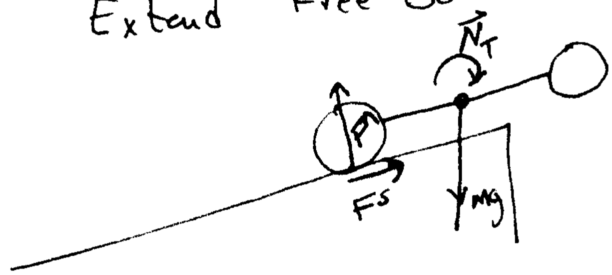
Extended Free Body



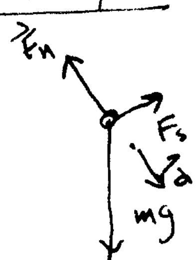
Phase II One Wheel on Ramp

Net Torque on bike, and an unbalanced force on the center of mass

Extend Free Body



Free Body



Phase III - Free Fall and Rotation Continue

Free Body

Extended Free Body

No torque