Homework 5

Due Tuesday 2/25/2003

Problem 1 Rouge physicists construct a Foucault's pendulum in Walton arena using 50m of rope and a bowling ball.

2 **(a)** What is the period of oscillation of the pendulum?

2 **(b)** What is the angular rate of precession?

2 **(c)** How long would it take for the pendulum to process back to its original orientation?

1 **(d)** What direction would it precess (clockwise or counterclockwise)?

0 **(e)** Would this be disruptive to the game?

Problem 2 An elevator accelerates upward at $g/8$. You toss your keys to Charlie who stands 2m away just before the acceleration begins. If the elevator was not accelerating, the keys would reach a maximum height above your hand of 0.25m. Charlie's hand is at the same height as your hand and the keys would have landed in his hand if the elevator had not been accelerating.

4 **(a)** Compute the initial velocity (v_{x0} and v_{z0}) of the keys?

2 **(b)** In the accelerating frame, what is the force on the keys?

3 **(c)** How much up or down do you miss by?

Problem 3

5 By what angle is a plumb line deflected in Fayetteville? What is the magnitude of the inertial force on the plumb bob at Fayetteville? (In terms of g)

Problem 4 The merry-go-round at Wilson park is about 2.5m in radius. I can spin it so it goes around once in 4 seconds. Kat walks radially outward (in the rotating frame) at normal Kat walking speed, $2\frac{m}{s}$.

6 **(a)** Compute the transverse, centrifugal, and Coriolis forces on Kat as a function of radius.

2 **(b)** Express the forces in (a) as a fraction of gravity.

3 **(c)** In the shoes she is wearing, she would begin to slip if the surface was inclined at 60°, so the coefficient of static friction is $\mu_s = \tan 60°$. Does she slip before she gets to the edge?

Problem 5 You throw a ball off the merry-go-round in the previous problem. In the fixed coordinate system, the ball travels in a trajectory

$$\vec{r}(t) = (v_{0x}t, 0, v_{0z}t - \frac{1}{2}gt^2)$$

4 (a) Transform the ball's trajectory to the frame rotating with the merry-go-round.

2 (b) Differentiate the trajectory to get the velocity in the rotating frame.

4 (c) Compute the velocity in the rotating frame $(\frac{d\vec{r}}{dt})_{rotating}$ using the relation in 5.2.9 on the fixed trajectory. Report the derivative using the fixed basis set $\hat{i}, \hat{j}, \hat{k}$.

4 (d) Transform the velocity to the frame rotating with the merry-go-round and compare with (b).

Problem 6 Home plate is 60ft from the pitcher's mound. The best pitchers throw at a speed of 100mph. The ball is thrown horizontally. Compute the deflection from the fixed earth path (Δx, Δy, Δz) due to the rotating earth if the line from the pitcher's mound to home plate points:

5 (a) North.

5 (b) East.

5 (c) North-East.

Find the time to traverse the 60ft by approximating the ball's path as a straight line.

Bonus Problem B.1 In BlackAdder III, what will the name of Edmond's future wife be?

+5

Catherine

5.1

(a) $\omega_0 = \sqrt{g/\ell} = \sqrt{\dfrac{9.81\,m/s^2}{50m}}$

$= 0.44\ s^{-1}$ $\quad T = \dfrac{2\pi}{\omega} = 14.17\ s$

(b) $\omega_p = \omega \sin\lambda$

$= \dfrac{2\pi}{1\,day} \sin 36^\circ$

$= 2\pi * 0.59\ (day)^{-1}$

(c) Period $= \dfrac{1}{0.59\ day^{-1}} = 1.7$ days $= \dfrac{2\pi}{\omega_p}$

(d) clockwise, we're in the northern hemisphere.

(e) You bet, but we're out of it this season.

5-2 $\vec{A}_o = g/8\ \hat{k}$

a

$$t_f = \frac{d}{V_{xo}}$$

$$h_{max} = 0.25m = \frac{V_{oz}^2}{2g}$$ (from previous hwk)

$$v_{oz} = \sqrt{2g\,h_{max}}$$

$$\boxed{v_{oz} = 2.21\ m/s}$$

$$t_{max} = \frac{v_{oz}}{g} = 0.23\,s$$

$$t_f = 0.45\,s$$

$$\boxed{V_{xo} = \frac{d}{t_f} = 4.43\ m/s}$$

(b) $\vec{F}_{physical} - m\vec{A}_o = m\vec{a}'$

$$-mg\hat{k} - m\,g/8\,\hat{k} = -m\frac{9}{8}g\,\hat{k} = F_{vse}$$

(c) Motion is still constant acceleration with

$$a = -\frac{9}{8}g$$

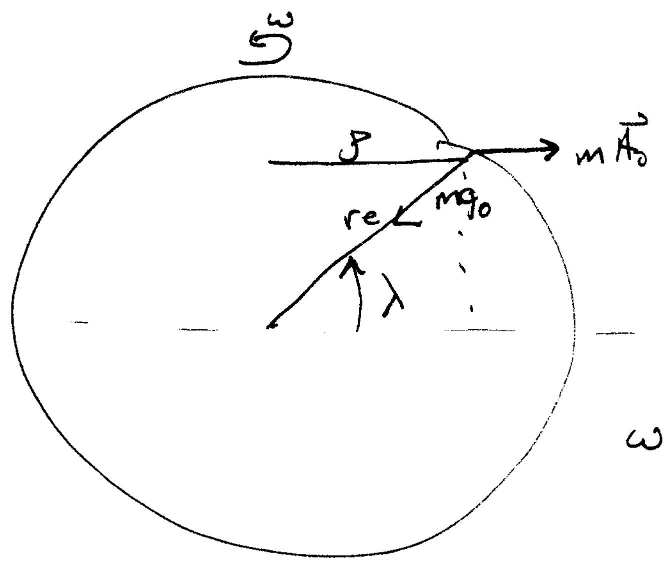
The time of flight is the same

$$h(t_f) = v_{0z}t_f - \frac{1}{2}\left(\frac{9}{8}\right)g\,t_f^2$$

$$= (2.21\,m/s)(0.45s) - \frac{1}{2}(9/8)(9.81\,m/s^2)(0.45s)^2$$

$$= \cancel{2.11}\,m \rightarrow 12\,cm$$

5.3



$$r_e = \frac{12{,}756{,}000 \text{ m}}{2} = 6.38 \times 10^{6} \text{ m}$$

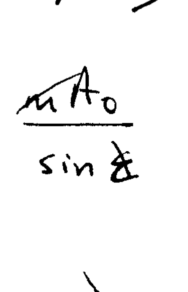
$$\omega = 7.27 \times 10^{-5} \text{ s}^{-1}$$

$$\text{Centripetal Acceleration} = \frac{v^2}{r} = \rho\omega^2$$

$$\rho = r_e \cos\lambda$$

$$v = \rho\omega = \cancel{r_e\omega^2 \cos^2\lambda}$$

$$A_0 = -r_e\omega^2\cos\lambda = \cancel{0.0\ldots} \approx \varepsilon g = 0.016 \text{ m/s}^2$$



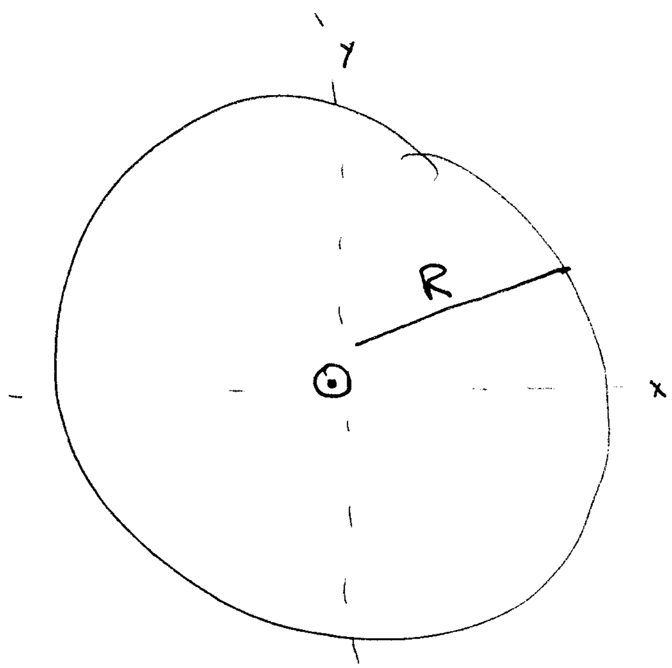
Law of Sines

$$\frac{mg}{\sin\lambda} = \frac{mA_0}{\sin\varepsilon}$$

$$\sin\varepsilon = \frac{r_e\omega^2\cos\lambda\sin\lambda}{g} = \frac{r_e\omega^2}{2g}\sin 2\lambda$$

$$\varepsilon = \sin^{-1}\left[\frac{r_e\omega^2}{2g}\sin 2(36^\circ)\right] = 0.94^\circ = 1.63\times10^{-3} \text{ rad}$$

5.4



$v_0 = 2\ m/s$

$R = 2.5\,m$

$T = 4\,s$

$\hat{k}$ out of page

$$\omega = \frac{2\pi}{T} = \frac{1}{2}\pi\,s = 1.57\,s^{-1}$$

(a) $\vec{\omega} = \omega\hat{k}$ Velocity $\vec{v}' = v_0\,\hat{\imath}'$

$$\vec{r}' = v_0 t\,\hat{\imath}'$$

$$\boxed{\text{Tranverse Force} = 0 \text{ because } \dot{\vec{\omega}} = 0.}$$

Centrifugal ~~Centripetal~~ Force $-m\vec{\omega}\times(\vec{\omega}\times\vec{r}') = \vec{F}_{cent}$

$$\vec{\omega}\times\vec{r}' = \omega v_0 t\,\hat{\jmath}' \qquad \text{(RHR)}$$

$$\vec{\omega}\times(\omega v_0 t\,\hat{\jmath}') = -\omega^2 v_0 t\,\hat{\imath}'$$

$$\boxed{\vec{F}_{cent} = m\omega^2 v_0 t\,\hat{\imath}'} = m\omega^2 r\,\hat{\imath}' = m\omega^2 x'\,\hat{\imath}'$$

$$\vec{F}_{cor} = -2m(\vec{\omega} \times \vec{v}\,')$$

$$\boxed{\vec{F}_{cor} = -2m\omega v_0 \hat{j}}$$

(b)

<u>Numerically</u> $\dfrac{|\vec{F}_{cor}|}{m} = +2\omega v_0 = 2\left(\tfrac{1}{2}\pi s^{-1}\right)\left(2\ {}^{m}/_{s}\right)$

$$= 2\pi\ {}^{m}/_{s^2}$$

$$= 6.28\ {}^{m}/_{s^2}$$

$$= 0.64g$$ wow!

$$\frac{|\vec{F}_{cent}|}{m} = \omega^2 v_0 t$$

At the edge $v_0 t = R$, so the maximum centripetal force equals, $\dfrac{|\vec{F}^{max}_{cent}|}{m} = \omega^2 R = \left(\tfrac{1}{2}\pi s^{-1}\right)^2 (2.5m)$

$$= 6.16\ {}^{m}/_{s^2} = 0.63g$$

(c)

At maximum, $\vec{F} = m(-0.63g\,\hat{i} - 0.64g\,\hat{j})$

$$|\vec{F}| > \mu_s mg = 1.73mg$$

then Kat will slip.

$$|\vec{F}| = mg\sqrt{(0.69)^2 + (0.63)^2} = \text{[illegible]}$$

$$= 0.88\, mg$$

So the kat doesn't slip.

5.5

(a) $\vec{r} = v_{0x} t\,\hat{\imath} + \left(v_{0z} t - \frac{1}{2} g t^2\right)\hat{k}$

$\hat{\imath}' = \cos\omega t\,\hat{\imath} + \sin\omega t\,\hat{\jmath}$

$\hat{\jmath}' = -\sin\omega t\,\hat{\imath} + \cos\omega t\,\hat{\jmath}$

Make coordinate axis rotate in correct direction

$\hat{\imath}\cdot\hat{\imath}' = \cos\omega t \qquad \hat{\imath}\cdot\hat{\jmath}' = -\sin\omega t$

$\hat{\jmath}\cdot\hat{\jmath}' = \cos\omega t \qquad \hat{\jmath}\cdot\hat{\imath}' = \sin\omega t$

$\vec{r} = A_x\,\hat{\imath}' + A_y\,\hat{\jmath}' + \left(v_{0z} t - \frac{1}{2} g t^2\right)\hat{k}$

$\vec{r}\cdot\hat{\imath}' = A_x = v_{0x} t\,(\hat{\imath}\cdot\hat{\imath}') = v_{0x} t \cos\omega t$

$\hat{r}\cdot\hat{\jmath}' = A_T = v_{0x} t\,(\hat{\imath}\cdot\hat{\jmath}') = -v_{0x} t \sin\omega t$

$$\boxed{\vec{r}(t) = v_{0x} t\cos\omega t\,\hat{\imath}' - v_{0x} t \sin\omega t\,\hat{\jmath}' + \left(v_{0z} t - \frac{1}{2} g t^2\right)\hat{k}'}$$

(b) $\left(\dfrac{d\vec{r}}{dt}\right)_{rot} = \left(v_{0x}\cos\omega t - v_0 t\omega \sin\omega t\right)\hat{\imath}'$

$- \left[v_0 \sin\omega t + v_{0x} t\omega\cos\omega t\right]\hat{\jmath}'$

$+ \left(v_{0z} - \cancel{2} g t\right)\hat{k}'$

(c) $$\left(\frac{d\vec{r}}{dt}\right)_{rot} = \left(\frac{d\vec{r}}{dt}\right)_{fixed} - \vec{\omega} \times \vec{r}$$

$$\left(\frac{d\vec{r}}{dt}\right)_{fixed} = v_{ox}\hat{\imath} + (v_{oz} - gt)\hat{k}$$

$$\vec{\omega} \times \vec{r} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & 0 & \omega \\ v_{ox}t & 0 & v_{oz}t - \frac{1}{2}gt^2 \end{vmatrix}$$

$$= + v_{ox} t \omega \hat{\jmath}$$

$$\boxed{\left(\frac{d\vec{r}}{dt}\right)_{rot} = v_{ox}\hat{\imath} - v_{ox} t \omega \hat{\jmath} + (v_{oz} - gt)\hat{k}}$$

(d) $$\left(\frac{d\vec{r}}{dt}\right)_{rot} = A_x \hat{\imath}' + A_y \hat{\jmath}' + (v_0 - gt)\hat{k}'$$

$$A_x = \hat{\imath}' \cdot \left(\frac{d\vec{r}}{dt}\right)_{rot} = v_{0x}\, \hat{\imath} \cdot \hat{\imath}' - v_{0x}\, t\omega\, \hat{\imath}' \cdot \hat{\jmath}$$

$$= v_{0x} \cos\omega t - v_{0x}\, t\omega \sin\omega t$$

$$A_y = \hat{\jmath}' \cdot \left(\frac{d\vec{r}}{dt}\right)_{rot} = v_{0x}\, \hat{\imath} \cdot \hat{\jmath}' - v_{0x}\, t\omega\, \hat{\jmath} \cdot \hat{\jmath}'$$

$$= -\sin\omega t\; v_{0x} - v_{0x}\, t\omega \cos\omega t$$

$$\vec{v}'(t) = \left(v_{0x} \cos\omega t - v_{0x}\, t\omega \sin\omega t\right) \hat{\imath}$$

$$- \left[v_{0x} \sin\omega t + v_{0x}\, t\omega \cos\omega t\right] \hat{\jmath}$$

$$+ \left(v_0 - gt\right) \hat{k}$$

Hozzah

5.6 Time in flight is $\dfrac{d}{V} = t$

$$d = 60\,\text{ft} \cdot \left(\frac{0.3048\,\text{m}}{1\,\text{ft}}\right) = 18.3\,\text{m}$$

$$V_0 = 100\,\text{mph}\left(\frac{0.447\,\text{m/s}}{1\,\text{mph}}\right) = 44.7\,\text{m/s}$$

$$t_f = \frac{18.3\,\text{m}}{44.7\,\text{m/s}} = 0.409\,\text{s}$$

(a) North is $\hat{j}'$

$$\boxed{\dot{x}_0' = 0 \;,\quad \dot{y}_0' = V_0}$$

In General

$$\Delta x = x'(t) - x_{\text{fixed}}(t) = \frac{1}{3}\omega g t^3 \cos\lambda \cancel{- \omega t \dot{z}_0'}$$
$$- \omega t^2\left(\dot{z}_0' \cos\lambda - \dot{y}_0' \sin\lambda\right)$$

$$\Delta y = -\omega \dot{x}_0' t^2 \sin\lambda$$

$$\Delta z = \omega \dot{x}_0' t^2 \cos\lambda$$

$\lambda = 36^0$

$\cos\lambda = 0.809$

$\sin\lambda = 0.588$

For $\dot{x}_0' = 0 \qquad \dot{y}_0' = v_0$

$\omega = 7.27 \times 10^{-5} s^{-1}$

$$\Delta x = \frac{1}{3}\omega g t^2 \cos\lambda + \omega t^2 v_0 \sin\lambda$$

$$= \omega t^2 \left[\frac{1}{3} g t \cos\lambda + v_0 \sin\lambda\right]$$

$$= (1.22 \times 10^{-5} s)[1.08 \, m/s + 26.3 \, m/s]$$

$$\Delta x = 3.34 \times 10^{-4} m = 0.33 \, mm$$

$$\Delta y = 0 \qquad \Delta z = 0$$

(b) $\dot{x}_0' = v_0 \hat{i}' \qquad \dot{y}_0' = 0 \qquad \dot{z}_0' = 0$

$$\Delta x = \frac{1}{3}\omega t^3 g \cos\lambda$$

$$= \frac{1}{3}(7.27 \times 10^{-5} s^{-1})(0.409 s)^3 (9.81 \, m/s^2) \, 0.809$$

$$= 1.32 \times 10^{-5} m = 0.01 \, mm$$

$$\Delta y = -\omega v_0 t^2 \sin\lambda = -3.19 \times 10^{-4} m$$

$$\Delta z = \omega v_0 t^2 \cos\lambda = 4.40 \times 10^{-4} m$$

(c) $\dot{x}_0 = \dfrac{v_0}{\sqrt{2}} = 31.6 \text{ m/s} = \dot{y}_0$

$$\Delta x = \frac{1}{3}\omega g t^3 \cos\lambda + \omega t^2 \dot{y}_0' \sin\lambda$$

$$= \omega t^2 \left[\frac{1}{3} g t \cos\lambda + \dot{y}_0 \sin\lambda\right]$$

$$= (1.22\times10^{-5}\,\text{s})\left[1.08 \text{ m/s} + 18.6 \text{ m/s}\right]$$

$$= 2.40\times10^{-4}\,\text{m}$$

$$\Delta y = -\omega \dot{x}_0' t^2 \sin\lambda = -2.25\times10^{-4}\,\text{m}$$

$$\Delta z = \cancel{2}\, \omega \dot{x}_0' t^2 \cos\lambda = 3.11\times10^{-4}\,\text{m}$$