Example Test 2

Justin's Version

CO_2 Molecule

A CO_2 molecule can be modeled as three masses connected by 2 springs of spring constant k. Let the mass of the carbon atom be $m_c = 3\,m$ and the mass of the oxygen atoms each be $m_O = 4\,m$ (even though these numbers may not look right, the ratio is correct). Allow only motions along the molecular axis (back and forth, not up and down).

a) Find the Lagrangian of this system.

b) Find the equations of motion of the system.

c) Find the Hamiltonian of the system.

d) Find the frequencies of the system.

e) Find the normal modes of the system. Are all these vibrations?

Stability

A particle of mass m moves in the following potential. Is there a stable orbit? If so, what is the frequency, ω, of the motion?

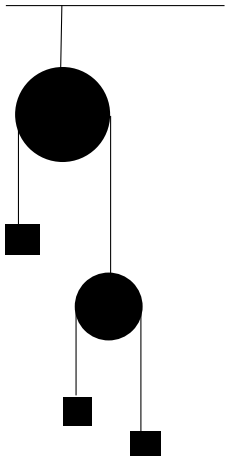
1) $V = k\,x^{-2}\,e^{x} + c$

2) $V = A\,\text{Cos}(k\,x + \pi/4)$

Double Atwood's Machine.

The following diagram shows a double Atwood's machine. Let each pulley be of mass M, each block be of mass m, and each string be of length l.
1) Use the Lagrangian to find the equations of motion for this system.

2)
Finding a Hamiltonian in terms of q_i and p_i is difficult for the double Atwood's Machine. Why might that be?



Forces of Constraint

A small, wet bar of soap of mass *m* can move about the inside of a hemispherical bowl of radius R.
1) Write the Lagrangian for this system.
2) What is the constraint for this system.
3) Write the equations of motion.
4) What is the normal force on the soap?