Quantum Mechanics Summer 2003- Homework Set 2

Due at beginning of class July 14, 2003.

Cohen-Tannoudji Problems - in H_{II}

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- 2 (Part a and b only)
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Additional Problems

Problem A1 Prove the following properties of the adjoint operation. (a) $(\hat{A}B)^\dagger = \hat{B}^\dagger \hat{A}^\dagger$ (b) $(\hat{A}^\dagger)^\dagger = \hat{A}$.

Problem A2 Prove that the set of square integrable functions is a vector space. Prove that the momentum operator

$$\hat{P} = \frac{\hbar}{i}\frac{\partial}{\partial x}$$

is Hermitian in this space.

Problem A3 Prove the following properties of the commutator of operators $\hat{A}$, $\hat{B}$, and $\hat{C}$.

(a) $[\hat{A}, \hat{B}\hat{C}] = [\hat{A}, \hat{B}]\hat{C} + \hat{B}[\hat{A}, \hat{C}]$

(b) $[\hat{A}, [\hat{B}, \hat{C}]] + [\hat{B}, [\hat{C}, \hat{A}]] + [\hat{C}, [\hat{A}, \hat{B}]] = 0$

(c) $[\hat{A}, \hat{B}]^\dagger = [\hat{B}^\dagger, \hat{A}^\dagger]$

Problem A4 Consider the position operator $\hat{X} = x$ and the momentum operator $\hat{P} = \frac{\hbar}{i}\frac{\partial}{\partial x}$. Show the following earth shattering relations.

(a) $[\hat{X}, \hat{P}] = i\hbar$

(b) $[\hat{X}, \hat{P}^2] = 2i\hbar P$

(c) Arguing by induction, you can show the first two results generalize to $[\hat{X}, \hat{P}^n] = i\hbar n P^{n-1}$. Use this result and the Taylor expansion to show.

$$[\hat{X}, F(\hat{P})] = i\hbar \frac{dF(P)}{dP}$$

where $F(P)$ is some function of P. It is also true but you do not have to show it that

$$[\hat{P}, G(\hat{X})] = -i\hbar \frac{dG(X)}{dX}$$

Problem A5 Prove that the product of two unitary operators is unitary.

Problem A6 Prove the following by examining the power series,

$$e^{ix} = \cos(x) + i\sin(x)$$

Griffith Problems

Problem G1 Consider the following matrices

$$\hat{A} = \begin{pmatrix} -1 & 1 & i \\ 2 & 0 & 3 \\ 2i & -2i & 2 \end{pmatrix}$$

$$\hat{B} = \begin{pmatrix} 2 & 0 & -i \\ 0 & 1 & 0 \\ i & 3 & 2 \end{pmatrix}$$

(a) Calculate $A + B$, AB, $[A, B]$, $\tilde{A}$, A^*, $A^\dagger$, $Tr(B)$, $det(B)$, and B^{-1}

(b) Does A have an inverse?

Problem G2 Let

$$\hat{T} = \begin{pmatrix} 1 & 1-i \\ 1+i & 0 \end{pmatrix}$$

(a) Verify that $\hat{T}$ is Hermitian.

(b) Find its eigenvalues (note they are real).

(c) Find the normalized eigenvectors.

(d) Construct the Unitary diagonalizing matrix, S^{-1}.

Problem G3 Consider the space of all polynomials up to order 3, called $P(3)$, with complex coefficients. Things that look like

$$P_3(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3$$

Let the inner product for these functions be defined as

$$< f_1 | f_2 >= \int_{-1}^{1} f_1^* f_2 dx$$

(a) Show that $P(3)$ is a vector space.

(b) Is the operator $i\frac{d}{dx}$ a linear operator in this space? How about x?

(c) Give a set of vectors that span the space.

(d) Construct an orthonormal basis(the Legendre Polynomials) using the Gram-Schmidt process.

(e) Compute the matrix representing the operator $i\frac{d}{dx}$ in your orthonormal basis.

(f) Is the operator in (e) Hermitian, why?