Quantum Mechanics Summer 2003- Homework Set 4

Due whenever. We will work on these together next week, so you will have 11 hours of my help to get through these. There are quite a number of parts, but by this time many of the parts should be something you've done before.

Simple Harmonic Oscillator

For these problems, use a simple harmonic oscillator with Hamiltonian

$$H = \frac{p^2}{2m} + \frac{1}{2}kx^2$$

and use $\omega = \sqrt{k/m}$.

Problem H1 Compute the commutation relation for the raising and lower operators $[\hat{a}_+, \hat{a}_-]$.

Problem H2 The following applies to a one dimensional SHO.

(a) Show the ground state wave function $\phi_0 = A\exp^{-ax^2}$ becomes zero when operated on by the lowering operator. From this deduce a.

(b) Compute the normalization constant, A.

(c) Construct the first excited state ϕ_1 by operating on the ϕ_0 with the raising operator.

(d) Consider the wavefunction $\psi = A\exp^{-2ax^2}$. This wave function is narrower than the ground state. Normalize the wave function and report the probability the system is in the ground state. The first excited state?(A little thought here will prevent integration).

Spin 1/2 Systems

Problem S1 Calculate the commutation relations for $\hat{S}_x$, $\hat{S}_y$, and $\hat{S}_z$.

Problem S2 Cohen-Tannoudji Chapter 4 Compliment J_{IV} Problem 1

Angular Momentum

Problem A1 Show $\hat{L}_x$, $\hat{L}_y$ satisfy the commutation relation for angular momentum.

$$[L_x, L_y] = i\hbar L_z$$

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Problem A2 Cohen-Tannoudji Chapter 6 Compliment F_{VI} Problem 1

Problem A3 Calculate the matrices representing J_x and J_z for $\ell = 1$.

Problem A4 Cohen-Tannoudji Chapter 6 Compliment F_{VI} Problem 2

Problem A5 Write the Y_1^1 and Y_0^0 spherical harmonics from the notes. Apply the normalized ladder operator to construct Y_0^1 and Y_{-1}^1

Problem A6 Cohen-Tannoudji Chapter 6 Compliment F_{VI} Problem 5

Three Dimensional Problems

Problem R1 A rigid rotor is two masses m_1 and m_2 connected by a rigid massless stick. The energy of the classical system is

$$E = \frac{L^2}{2I}$$

where I is the moment of inertia about the center of mass and $\vec{L}$ is the angular momentum. Quantize the system. Find the eigenvalues and eigenvectors of the Hamiltonian. What is the ground state energy of the rigid rotor? The first excited state energy?

Problem R2 Consider a spherical infinite potential well where $V = 0$ if $r < a$ and $V = \infty$ if $r > a$.

(a) Write the Hamiltonian of the system in spherical coordinates.

(b) Separate the Hamiltonian into a radial and angular parts.

(c) Solve the angular part. The answer is always the same for a radial potential.

(d) Write the differential equation the radial part satisfies.

(e) Find the solution to the differential equation in (d) in a math table.

(f) Write the ground state wave function and energy. The wave function may be in terms of an appropriate special function.

Problem R3 Consider hydrogen atom where the proton is assumed fixed at the origin.

(a) Write the Hamiltonian of the system in spherical coordinates.

(b) Separate the Hamiltonian into a radial and angular parts.

(c) Solve the angular part. The answer is always the same for a radial potential.

(d) Write the differential equation the radial part satisfies.

(e) Look up and report the $|n, \ell, m>$ wave functions for $|1, 0, 0>$, the ground state and the $|2, 1, -1>$.

(f) Calculate the expectation value of r for the ground state. This is the Bohr radius, the average distance of the electron from the nucleus.