PHYS 4073 - Quantum Mechanics- Homework Set 10

Reading Assignment: Chapter 5

Due Friday December 3rd at the beginning of class.

Turn in seven of the following problems.

Griffiths' Problems

4.27

4.32

4.35

4.36

4.49

4.52

4.55

5.6

5.7

5.12

5.32

5.33

4.27

$$|\psi\rangle = A\begin{pmatrix} 3i \\ 4 \end{pmatrix}$$

$$\langle\psi|\psi\rangle = A^*A(9+16) = 1$$

$$A = \frac{1}{5}$$

$$|\psi\rangle = \frac{1}{5}\begin{pmatrix} 3i \\ 4 \end{pmatrix}$$

(b) $$\langle\psi|S_x|\psi\rangle = \frac{1}{25}\frac{\hbar}{2}(-3i, 4)\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 3i \\ 4 \end{pmatrix}$$

$$= \frac{\hbar}{50}(-3i, 4)\begin{pmatrix} 4 \\ 3i \end{pmatrix}$$

$$= 0$$

$$\langle\psi|S_y|\psi\rangle = \frac{\hbar}{50}(-3i, 4)\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 3i \\ 4 \end{pmatrix}$$

$$= \frac{\hbar}{50}(-3i, 4)\begin{pmatrix} -4i \\ -3 \end{pmatrix}$$

$$= \frac{\hbar}{50}(-12 - 12) = -\frac{2}{25}\hbar$$

$$\langle S_x \rangle = \frac{\hbar}{50}\begin{pmatrix} -3i & 4 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 3i \\ 4 \end{pmatrix}$$

$$= \frac{\hbar}{50}\begin{pmatrix} -3i, & 4 \end{pmatrix}\begin{pmatrix} 3i \\ -4 \end{pmatrix}$$

$$= \frac{\hbar}{50}(9-16) = \frac{-7\hbar}{50}$$

(c)

$$\sigma_x^2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\sigma_y^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\sigma_z^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\langle S_x^2 \rangle = \langle S_y^2 \rangle = \langle S_z^2 \rangle = \frac{\hbar^2}{4}\langle \psi | 1 | \psi \rangle$$

$$= \frac{\hbar^2}{4}$$

$$\sigma_{S_x} = \sqrt{\langle S_x^2 \rangle - \langle S_x \rangle^2} = \frac{\hbar}{2}$$

$$\sigma_{S_y} = \sqrt{\langle S_y^2 \rangle - \langle S_y \rangle^2} = \sqrt{\frac{\hbar^2}{4} - \left(\frac{12}{2\cdot\cdot}\right)^2 \hbar^2}$$

$$= \frac{7}{50}\hbar$$

$$\sigma_{S_z} = \sqrt{\langle S_z^2 \rangle - \langle S_z \rangle^2}$$

$$= \sqrt{\frac{\hbar^2}{4} - \left(\frac{7}{50}\right)^2 \hbar^2} = \frac{12}{25}\hbar$$

(d) $\sigma_{S_x}\sigma_{S_y} \geq \frac{\hbar}{2} |\langle \psi | S_z | \psi \rangle|$

$\left(\frac{\hbar}{2}\right)\left(\frac{7\hbar}{50}\right) \geq \frac{\hbar}{2}\left(\frac{7\hbar}{50}\right)$ ✓ barely

$\sigma_{S_x}\sigma_{S_z} \geq \frac{\hbar}{2}|\langle S_y \rangle|$

$\left(\frac{\hbar}{2}\right)\left(\frac{12\hbar}{25}\right) \geq \frac{\hbar}{2}\left(\frac{12}{25}\hbar\right)$ ✓

✓

$\sigma_{S_y}\sigma_{S_z} \geq 0$

4.32

$$\omega = \gamma B_0$$

$$|\psi\rangle = \cos\frac{\alpha}{2}\, e^{i\omega t/2}|+\rangle + \sin\frac{\alpha}{2}\, e^{-i\omega t/2}|-\rangle$$

$$|+\rangle_x = \frac{1}{\sqrt{2}}\left(|+\rangle + |-\rangle\right)$$

$$c_+ = {}_x\langle +|\psi\rangle = \frac{1}{\sqrt{2}}\left(\cos\frac{\alpha}{2}\, e^{i\omega t/2} + \sin\frac{\alpha}{2}\, e^{-i\omega t/2}\right)$$

$$= \frac{1}{\sqrt{2}}\, e^{i\omega t/2}\left(\cos\frac{\alpha}{2} + \sin\frac{\alpha}{2}\, e^{-i\omega t}\right)$$

$$\mathcal{P}\left(\frac{\pi}{2}\right) = c_+^* c_+ = \frac{1}{2}\left(\cos\frac{\alpha}{2} + \sin\frac{\alpha}{2}\, e^{i\omega t}\right)\left(\cos\frac{\alpha}{2} + \sin\frac{\alpha}{2}\, e^{-i\omega t}\right)$$

$$= \frac{1}{2}\left(\cos^2\frac{\alpha}{2} + \sin^2\frac{\alpha}{2} + \cos\frac{\alpha}{2}\sin\frac{\alpha}{2}\left(e^{i\omega t} + e^{-i\omega t}\right)\right)$$

$$= \frac{1}{2}\left(1 + \sin\alpha\cos\omega t\right)$$

(b) $|+\rangle_y = \frac{1}{\sqrt{2}}\left(|+\rangle + i|-\rangle\right)$

$$c_+ = {}_y\langle + | \psi \rangle = \frac{1}{\sqrt{2}}\left(\cos\frac{\alpha}{2} e^{i\omega t/2} + i\sin\frac{\alpha}{2} e^{-i\omega t/2}\right)$$

$$= \frac{1}{\sqrt{2}} e^{i\omega t/2}\left(\cos\frac{\alpha}{2} + i\sin\frac{\alpha}{2} e^{-i\omega t}\right)$$

$$P\left(\frac{\hbar}{2}\right) = c_+^* c_+ = \left(\frac{1}{\sqrt{2}}\right)^2\left(\cos\frac{\alpha}{2} - i\sin\frac{\alpha}{2} e^{i\omega t}\right)\left(\cos\frac{\alpha}{2} + i\sin\frac{\alpha}{2} e^{-i\omega t}\right)$$

$$= \frac{1}{2}\left(\cos^2\frac{\alpha}{2} + \sin^2\frac{\alpha}{2} - i\cos\frac{\alpha}{2}\sin\frac{\alpha}{2}\left(e^{-i\omega t} - e^{i\omega t}\right)\right)$$

$$= \frac{1}{2}\left(1 + \sin\alpha\,\frac{\left(e^{-i\omega t} - e^{i\omega t}\right)}{2i}\right)$$

$$= \frac{1}{2}\left(1 - \sin\alpha \sin\omega t\right)$$

(c) $|+\rangle_z = |+\rangle$

$$c_+ = \langle + | \psi \rangle = \cos\frac{\alpha}{2} e^{i\omega t/2}$$

$$P\left(\frac{\hbar}{2}\right) = c_+^* c_+ = \cos^2\frac{\alpha}{2}$$

4.35

3 spin $1/2$ particles = baryon

Add two spin $\frac{1}{2}$ and you get spin 1 or spin 0

spin 1 + ~~spin~~ spin $1/2$ $\longrightarrow |s_1 + s_2| \ldots |s_1 - s_2|$

$= 3/2 \ldots |1 - 1/2| = 1/2$

$\Rightarrow$ spin $3/2$ or $1/2$

Meson = 2 spin $1/2$

= spin 1 or spin 0

4.36

$$|3\,1\rangle = \sum |1\,?\rangle|2\,?\rangle$$

Look in 2×1 table Clebsch-Gordan, 3, 1

column.

$$|3\,1\rangle = \sqrt{\tfrac{1}{15}}\,|2\,2\rangle|1\,{-1}\rangle + \sqrt{\tfrac{8}{15}}\,|2\,1\rangle|1\,0\rangle + \sqrt{\tfrac{6}{15}}\,|2\,0\rangle|1\,1\rangle$$

Measure S_z of spin 2 particle

$$P(2\hbar) = \frac{1}{15} \qquad P(\hbar) = \frac{8}{15} \qquad P(0) = \frac{6}{15}$$

(b) Total angular momentum is $\vec{L} + \vec{S}$

Orbital angular momentum $l = 1$, $m = 0$

Spin $s = 1/2$ $m = -1/2$

$$|1\,0\rangle|\tfrac{1}{2}\,{-\tfrac{1}{2}}\rangle = \sqrt{\tfrac{2}{3}}\,|\tfrac{3}{2}\,{-\tfrac{1}{2}}\rangle + \sqrt{\tfrac{1}{3}}\,|\tfrac{1}{2}\,{-\tfrac{1}{2}}\rangle$$

$$P\left(j = 3/2 \rightarrow J = \tfrac{3}{2}\left(\tfrac{3}{2}+1\right)\hbar^2 = \tfrac{15}{4}\hbar^2\right) = \frac{2}{3}$$

$$P\left(j = 1/2 \rightarrow J = \tfrac{3}{4}\hbar^2\right) = \frac{1}{3}$$

4.49

$$|\psi\rangle = A\begin{pmatrix} 1-2i \\ 2 \end{pmatrix}$$

$$\langle\psi|\psi\rangle = AA^*\left((1-2i)(1+2i)+4\right)$$

$$= AA^*(1+4+4) = 9$$

$$A = \frac{1}{3}$$

(b) Measure S_z

$$\mathcal{P}\left(\frac{\hbar}{2}\right) = |\langle +|\psi\rangle|^2 = \frac{5}{9}$$

$$\mathcal{P}\left(-\frac{\hbar}{2}\right) = |\langle -|\psi\rangle|^2 = \frac{4}{9}$$

Expectation value $= \frac{5}{9}\cdot\left(\frac{\hbar}{2}\right) + \frac{4}{9}\left(-\frac{\hbar}{2}\right) = \frac{\hbar}{18}$

(c) $|+\rangle_x = \frac{1}{\sqrt{2}}\left(|+\rangle + |-\rangle\right)$

$$\mathcal{P}\left(\frac{\hbar}{2}\right) = |{}_x\langle +|\psi\rangle|^2 = \frac{1}{9\cdot 2}|1-2i+2|^2$$

$$= \frac{1}{2}\cdot\frac{1}{9}(9+4) = \frac{13}{18}$$

$$\mathcal{P}\left(-\frac{\hbar}{2}\right) = 1 - \mathcal{P}\left(\frac{\hbar}{2}\right) = \frac{5}{18}$$

Expectation Value $\langle S_x\rangle = \frac{\hbar}{2}\cdot\frac{13}{18} + \left(-\frac{\hbar}{2}\right)\frac{5}{18} = \frac{8}{36}\hbar = \frac{2}{9}\hbar$

(d) $|+\rangle_y = \frac{1}{\sqrt{2}}\left(|+\rangle + i|-\rangle\right)$

$${}_y\langle +|\psi\rangle = \frac{1}{3\sqrt{2}}(1\ ,\ -i)\begin{pmatrix} 1-2i \\ 2 \end{pmatrix}$$

$$= \frac{1}{3\sqrt{2}}\left(1 - 2i - 2i\right)$$

$$= \frac{1-4i}{3\sqrt{2}}$$

$P\left(\frac{\hbar}{2}\right) = \frac{17}{18}$ $\qquad$ $P\left(-\frac{\hbar}{2}\right) = \frac{1}{18}$

$$\langle S_y\rangle = \frac{\hbar}{2}\cdot\frac{17}{18} + \left(-\frac{\hbar}{2}\right)\frac{1}{18} = \frac{16}{18}\frac{\hbar}{2} = \frac{4\hbar}{9}$$

4.52 For spin $3/2$, the basis states are

$$\left|\tfrac{3}{2}\ \tfrac{3}{2}\right\rangle \quad \left|\tfrac{3}{2}\ \tfrac{1}{2}\right\rangle \quad \left|\tfrac{3}{2}\ -\tfrac{1}{2}\right\rangle \quad \left|\tfrac{3}{2}\ -\tfrac{3}{2}\right\rangle$$

$$\equiv \quad \equiv \quad \equiv \quad \equiv$$

$$\left|\tfrac{3}{2}\right\rangle \quad \left|\tfrac{1}{2}\right\rangle \quad \left|-\tfrac{1}{2}\right\rangle \quad \left|-\tfrac{3}{2}\right\rangle$$

$$S_x = \begin{pmatrix} \langle \tfrac{3}{2}|S_x|\tfrac{3}{2}\rangle & \langle \tfrac{3}{2}|S_x|\tfrac{1}{2}\rangle & \langle \tfrac{3}{2}|S_x|-\tfrac{1}{2}\rangle & \langle \tfrac{3}{2}|S_x|-\tfrac{3}{2}\rangle \\ \boxed{\langle \tfrac{1}{2}|S_x|\tfrac{3}{2}\rangle} & \langle \tfrac{1}{2}|S_x|\tfrac{1}{2}\rangle & \langle \tfrac{1}{2}|S_x|-\tfrac{1}{2}\rangle & \langle \tfrac{1}{2}|S_x|-\tfrac{3}{2}\rangle \\ \langle -\tfrac{1}{2}|S_x|\tfrac{3}{2}\rangle & \boxed{\langle -\tfrac{1}{2}|S_x|\tfrac{1}{2}\rangle} & \langle -\tfrac{1}{2}|S_x|-\tfrac{1}{2}\rangle & \langle -\tfrac{1}{2}|S_x|-\tfrac{3}{2}\rangle \\ \langle -\tfrac{3}{2}|S_x|\tfrac{3}{2}\rangle & \langle -\tfrac{3}{2}|S_x|\tfrac{1}{2}\rangle & \boxed{\langle -\tfrac{3}{2}|S_x|-\tfrac{1}{2}\rangle} & \langle -\tfrac{3}{2}|S_x|-\tfrac{3}{2}\rangle \end{pmatrix}$$

Only elements that are 1 apart are non-zero.

$$S_x = \frac{1}{2}(S_+ + S_-)$$

$$S_-\left|\tfrac{3}{2}\ \tfrac{3}{2}\right\rangle = \hbar\sqrt{\tfrac{3}{2}\left(\tfrac{3}{2}+1\right) - \tfrac{3}{2}\left(\tfrac{3}{2}-1\right)}\ \left|\tfrac{3}{2}\ \tfrac{1}{2}\right\rangle$$

$$= \hbar\sqrt{\frac{15}{4} - \frac{3}{4}}\ \left|\tfrac{1}{2}\right\rangle$$

$$= \frac{\hbar}{2}\sqrt{12}\ \left|\tfrac{1}{2}\right\rangle = \hbar\sqrt{3}\ \left|\tfrac{1}{2}\right\rangle$$

$$S_- \left| \tfrac{3}{2}\ \tfrac{1}{2} \right\rangle = \hbar \sqrt{\tfrac{3}{2}\left(\tfrac{3}{2}+1\right) - \tfrac{1}{2}\left(\tfrac{1}{2}-1\right)} \left| -\tfrac{1}{2} \right\rangle$$

$$= \hbar \sqrt{\frac{15}{4} + \frac{1}{4}} \left| -\tfrac{1}{2} \right\rangle$$

$$= 2\hbar \left| -\tfrac{1}{2} \right\rangle$$

$$S_- \left| \tfrac{3}{2}\ -\tfrac{1}{2} \right\rangle = \hbar \sqrt{\tfrac{3}{2}\left(\tfrac{3}{2}+1\right) + \tfrac{1}{2}\left(-\tfrac{1}{2}-1\right)} \left| -\tfrac{3}{2} \right\rangle$$

$$= \hbar \sqrt{\frac{15}{4} - \frac{3}{4}} \left| -\tfrac{3}{2} \right\rangle = \sqrt{3}\,\hbar \left| -\tfrac{3}{2} \right\rangle$$

$$S_x = \frac{\hbar}{2} \begin{pmatrix} 0 & \sqrt{3} & 0 & 0 \\ \sqrt{3} & 0 & 2 & 0 \\ 0 & 2 & 0 & \sqrt{3} \\ 0 & 0 & \sqrt{3} & 0 \end{pmatrix}$$

4.55

(a) Only $L^2 = 1(1+1)\hbar^2 = 2\hbar^2$ can be observed.

(b) $P(\hbar) = 2/3 \qquad P(0) = \frac{1}{3}$

(c) $P(\frac{3}{4}\hbar^2) = 1$

(d) $P(\frac{\hbar}{2}) = \frac{1}{3} \qquad P(-\frac{\hbar}{2}) = 2/3$

(e)

$$Y_1^0|+\rangle = |1\,0\rangle|\tfrac{1}{2}\,\tfrac{1}{2}\rangle = \sqrt{\tfrac{2}{3}}\,|\tfrac{3}{2}\,+\tfrac{1}{2}\rangle - \sqrt{\tfrac{1}{3}}\,|\tfrac{1}{2}\,\tfrac{1}{2}\rangle$$

$$\cancel{|1\,0\rangle|\tfrac{1}{2}\,-\tfrac{1}{2}\rangle = \sqrt{\tfrac{2}{3}}\,|\tfrac{3}{2}\,-\tfrac{1}{2}\rangle + \sqrt{\tfrac{1}{3}}\,|\tfrac{1}{2}\,-\tfrac{1}{2}\rangle}$$

$$Y_1^1|-\rangle = |1\,1\rangle|\tfrac{1}{2}\,-\tfrac{1}{2}\rangle = \sqrt{\tfrac{1}{3}}\,|\tfrac{3}{2}\,\tfrac{1}{2}\rangle + \sqrt{\tfrac{2}{3}}\,|\tfrac{1}{2}\,\tfrac{1}{2}\rangle$$

$$|\psi\rangle = R_{21}\Big[\Big(\sqrt{\tfrac{1}{3}}\Big)\Big(\sqrt{\tfrac{2}{3}}\,|\tfrac{3}{2}\,\tfrac{1}{2}\rangle - \sqrt{\tfrac{1}{3}}\,|\tfrac{1}{2}\,\tfrac{1}{2}\rangle\Big)$$
$$\Big(\sqrt{\tfrac{2}{3}}\Big)\Big(\sqrt{\tfrac{1}{3}}\,|\tfrac{3}{2}\,\tfrac{1}{2}\rangle + \sqrt{\tfrac{2}{3}}\,|\tfrac{1}{2}\,\tfrac{1}{2}\rangle\Big)\Big]$$

$$|\psi\rangle = R_{21}\left[\frac{2\sqrt{2}}{3} \left| \tfrac{3}{2}\ \tfrac{1}{2} \right\rangle + \frac{1}{3} \left| \tfrac{1}{2}\ \tfrac{1}{2} \right\rangle \right]$$

$$\mathcal{P}\left(j = 3/2 \rightarrow J^2 = \tfrac{15}{4}\hbar^2 \right) = \frac{8}{9}$$

$$\mathcal{P}\left(j = 1/2 \rightarrow J^2 = \tfrac{3}{4}\hbar^2 \right) = \frac{1}{9}$$

(f) $\mathcal{P}\left(m = 1/2 \Rightarrow J_z = \frac{\hbar}{2} \right) = 1.$

(g)

$$p(x) = \psi^* \psi$$

$$= R_{21}^* R_{21} \left(\frac{1}{3} Y_1^{0*} Y_1^0 + \frac{2}{3} Y_1^{1*} Y_1^1 \right)$$

Since $\langle + | + \rangle = 1$, $\langle + | - \rangle = 0$

$$p(r) = \left(\frac{1}{24 a^5} r^2 e^{-r/a} \right) \left(\frac{1}{4\pi} \cos^2\theta + \frac{1}{4\pi} \sin^2\theta \right)$$

$$= \frac{1}{96 \pi a^5} r^2 e^{-r/a}$$

(h) Only the first term contributes for spin up

$$p(r) = \frac{1}{96 \pi a^5} r^2 e^{-r/a} \cos^2\theta$$

but we need to in out θ.

$$p(r) = \frac{1}{3} R_{21}^* R_{21} = \frac{1}{72 a^5} r^2 e^{-r/a}$$

using the fact that Y_1^0 is normalized.

5.7

Distinguishable

$$\psi = \phi_a(x_1)\,\phi_b(x_2)\,\phi_c(x_3)$$

Bosons Use Slater determinant but ignore negative signs

$$\psi = \frac{1}{\sqrt{6}} \det \begin{pmatrix} \phi_a(x_1) & \phi_b(x_1) & \phi_c(x_1) \\ \phi_a(x_2) & \phi_b(x_2) & \phi_c(x_2) \\ \phi_a(x_3) & \phi_b(x_3) & \phi_c(x_3) \end{pmatrix}$$

$$= \frac{1}{\sqrt{6}} \Big[\phi_a(x_1)\big[\phi_b(x_2)\phi_c(x_3) + \phi_c(x_2)\phi_b(x_3)\big]$$

$$+ \phi_b(x_1)\big[\phi_a(x_2)\phi_c(x_3) + \phi_c(x_2)\phi_a(x_3)\big]$$

$$+ \phi_c(x_1)\big[\phi_a(x_2)\phi_b(x_3) + \phi_b(x_2)\phi_a(x_3)\big]\Big]$$

$$= \frac{1}{\sqrt{6}} \Big[\phi_{a1}\phi_{b2}\phi_{c3} + \phi_{a1}\phi_{c2}\phi_{b3} + \phi_{b1}\phi_{a2}\phi_{c3}$$

$$+ \phi_{b1}\phi_{a2}\phi_{c3} + \phi_{c1}\phi_{a2}\phi_{b3} + \phi_{c1}\phi_{b2}\phi_{a3} \Big]$$

using $\phi_a(x_1) \equiv \phi_{a1}$

(c) Now take the determinant for real

$$\psi = \frac{1}{\sqrt{6}} \det \begin{pmatrix} \phi_{a1} & \phi_{b1} & \phi_{c1} \\ \phi_{a2} & \phi_{b2} & \phi_{c2} \\ \phi_{a3} & \phi_{b3} & \phi_{c3} \end{pmatrix}$$

$$= \frac{1}{\sqrt{6}} \Big(\phi_{a1}\phi_{b2}\phi_{c3} - \phi_{a1}\phi_{c2}\phi_{b3} - \phi_{b1}\phi_{a2}\phi_{c3}$$

$$+ \phi_{b1}\phi_{c2}\phi_{a3} + \phi_{c1}\phi_{a2}\phi_{b3} - \phi_{c1}\phi_{b2}\phi_{a3} \Big)$$

(5.9) The spectrum of helium is $Z^2 = 4$ times the spectrum of hydrogen

$$E_n = \frac{-4 \cdot 13.6\,eV}{n^2}$$

The first transition is from $2 \to 1$ with energy

$$\Delta E = -4 \cdot 13.6\,eV \left(\frac{1}{1^2} - \frac{1}{2^2}\right)$$

$$= 3 \cdot 13.6\,eV = 40.8\,eV$$

The energy to ionize the second electron is

$$E_2 = \frac{-4 \cdot 13.6\,eV}{2^2} = 13.6\,eV$$

The kinetic energy of the ionized particle is

$$\Delta E - E_2 = 27.2\,eV$$

(b) The spectrum of He^+ is the same as hydrogen with all transition energies multiplied by 4

$$\Delta E_{nm} = 4 \Delta E_{nm,\,Hydrogen}.$$

(5.12)

Z	element	
1	hydrogen	$1s^1$
2	helium	$1s^2$
3	lithium	$1s^2 2s^1$
4	Be	$1s^2 2s^2$
5	B	$1s^2 2s^2 2p^1$
6	C	$1s^2 2s^2 2p^2$
7	N	$1s^2 2s^2 2p^3$
8	O	$1s^2 2s^2 2p^4$
9	F	$1s^2 2s^2 2p^6$
10	Ne	$1s^2 2s^2 2p^6$

(b) hydrogen $S = \frac{1}{2}$ $L = 0$ $J = \frac{1}{2}$

$$^2S_{1/2}$$

helium - Must be anti-symmetric, $L = 0$

so $S = 0$ (singlet state), $J = 0$

$$^1S_0$$

<u>lithium</u> same reasoning, L, S of closed shell zero

$S = 1/2$, $L = 0$, $J = 1/2$

$${}^2S_{1/2}$$

<u>beryllium</u> Again same reasoning

$${}^1S_0$$

---

boron, carbon, and nitrogen have closed 1s and 2s shells that do not contribute. Only the p electrons matter.

<u>boron</u> 1 p electron

$S = 1/2$ $L = 1$ $J = 3/2, 1/2$

$${}^2P_{1/2,\,3/2}$$

<u>Carbon</u> 2 p electrons

$S = 1, 0$ $\quad$ $L = 2, 1, 0$

$^3D_{3,2,1}$ $\quad$ $^1D_2$ $\quad$ $^3P_{2,1,0}$ $\quad$ $^1P_1$ $\quad$ $^3S_1$ $\quad$ $^1S_0$

<u>nitrogen</u> 3 p electron

$S = 3/2, 1/2$ $\quad$ $L = 3, 2, 1, 0$

$^4F_{9/2, 7/2, 5/2, 3/2}$ $\quad$ $^2F_{7/2, 5/2}$

$^4D_{7/2, 5/2, 3/2, 1/2}$ $\quad$ $^2D_{5/2, 3/2}$

$^4P_{5/2, 3/2, 1/2}$ $\quad$ $^2P_{3/2, 1/2}$

$^4S_{3/2}$ $\quad$ $^2S_{1/2}$

Note, not all of the above states are consistent with the symmetrization rule.

5.32

$$\phi_a = A\, e^{-\eta^2/2}$$

$$\phi_b = \sqrt{2}\, A\, \eta\, e^{-\eta^2/2}$$

$$A \equiv \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \qquad \eta = \sqrt{\frac{m\omega}{\hbar}}\, x \qquad x = \sqrt{\frac{\hbar}{m\omega}}\, \eta$$

$$\langle x^2\rangle_a = A^2 \int x^2 e^{-\eta^2} dx$$

$$= A^2 \left(\frac{\hbar}{m\omega}\right)^{3/2} \int_{-\infty}^{\infty} \eta^2 e^{-\eta^2} d\eta$$

$$= \frac{\hbar}{2m\omega}$$

$$\langle x^2\rangle_b = 2A^2 \int x^2 \eta^2 e^{-\eta^2} dx$$

$$= 2A^2 \left(\frac{\hbar}{m\omega}\right)^{3/2} \int \eta^4 e^{-\eta^2} d\eta$$

$$= \frac{3\hbar}{2m\omega}$$

$$\langle x\rangle_a = \langle x\rangle_b = 0$$

$$\langle x \rangle_{ab} = A^2\sqrt{2} \int x\, \eta\, e^{-\eta^2} dx$$

$$= A^2\sqrt{2}\left(\frac{\hbar}{m\omega}\right) \int_{-\infty}^{\infty} \eta^2 e^{-\eta^2} d\eta$$

$$= \sqrt{\frac{\hbar}{2m\omega}}$$

(a) Distinguishable

$$\langle (x_1 - x_2)^2 \rangle = \langle x^2 \rangle_a + \langle x^2 \rangle_b - 2\langle x \rangle_a \langle x \rangle_b$$

$$= \frac{\hbar}{2m\omega} + \frac{3\hbar}{2m\omega} = \frac{2\hbar}{m\omega}$$

(b) Bosons

$$\langle (x_1 - x_2)^2 \rangle = \langle (x_1 - x_2)^2 \rangle_d - 2 \left| \langle x \rangle_{ab} \right|^2$$

$$= \frac{2\hbar}{m\omega} - 2\left(\frac{\hbar}{2m\omega}\right)$$

$$= \frac{\hbar}{m\omega}$$

(c) Fermions

$$\langle (x_1 - x_2)^2 \rangle = \langle (x_1 - x_2)^2 \rangle_d + 2 \left| \langle x \rangle_{ab} \right|^2$$

$$= \frac{3\hbar}{m\omega}$$

5.33

(a) Each particle can be in any of three states $N = 3^3 = 27$ states.

(b) Bosons

I. Each particle in 3 different states abc $= 1$

II. Two particles in same state

aab, aac, bba, bbc, cca, ccb

$=$ 6 states.

III All in the same state

aaa, bbb, ccc $=$ 3 states

$N = 10$ total states.

(c) Fermions All particles must be in different states.

$N = 1$