A1

$$V = e\,\Delta V$$

$$= 1.602\times10^{-19}\,C \cdot 3000\,V$$

$$= 4.8\times10^{-16}\,J$$

$$R = \left(\frac{k_1 - k_2}{k_1 + k_2}\right)^2 = 0.5$$

$$k_1 = \sqrt{\frac{2mE}{\hbar^2}} \qquad k_2 = \sqrt{\frac{2m(E-V)}{\hbar^2}}$$

Note, $E > V$ otherwise $R = 1$.

$$R = \left(\frac{\sqrt{E} - \sqrt{E-V}}{\sqrt{E} + \sqrt{E-V}}\right)^2 = \frac{1}{2}$$

$$\frac{E + E - V - 2\sqrt{E}\sqrt{E-V}}{E + E - V + 2\sqrt{E}\sqrt{E-V}} = 1/2$$

$$2E - V - 2\sqrt{E}\sqrt{E-V} = E - \frac{V}{2} + \sqrt{E}\sqrt{E-V}$$

$$E - \frac{V}{2} = 3\sqrt{E}\sqrt{E-V}$$

$$E^2 - EV + \frac{V^2}{4} = 9E(E-V) = 9E^2 - 9EV$$

$$8E^2 - 8EV - V^2/4 = 0$$

$$32E^2 - 32EV - V^2 = 0$$

Quadratic Formula

$$E = \frac{32V \pm \sqrt{1024V^2 + 128V^2}}{64}$$

$$= \frac{1}{2}V \pm \frac{V}{64} \cdot 8\sqrt{16 + 2}$$

$$= \frac{1}{2}V \pm \frac{3V}{8} \cdot \sqrt{2}$$

Select Positive Root $(E>0)$

$$E = \left(\frac{1}{2} + \frac{3\sqrt{2}}{8}\right)V = 1.03\,V = 4.95\times10^{-16}\,J$$

Speed (hope classical)

$$E = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2\cdot 4.95\times10^{-16}\,J}{9.11\times10^{-31}\,kg}}$$

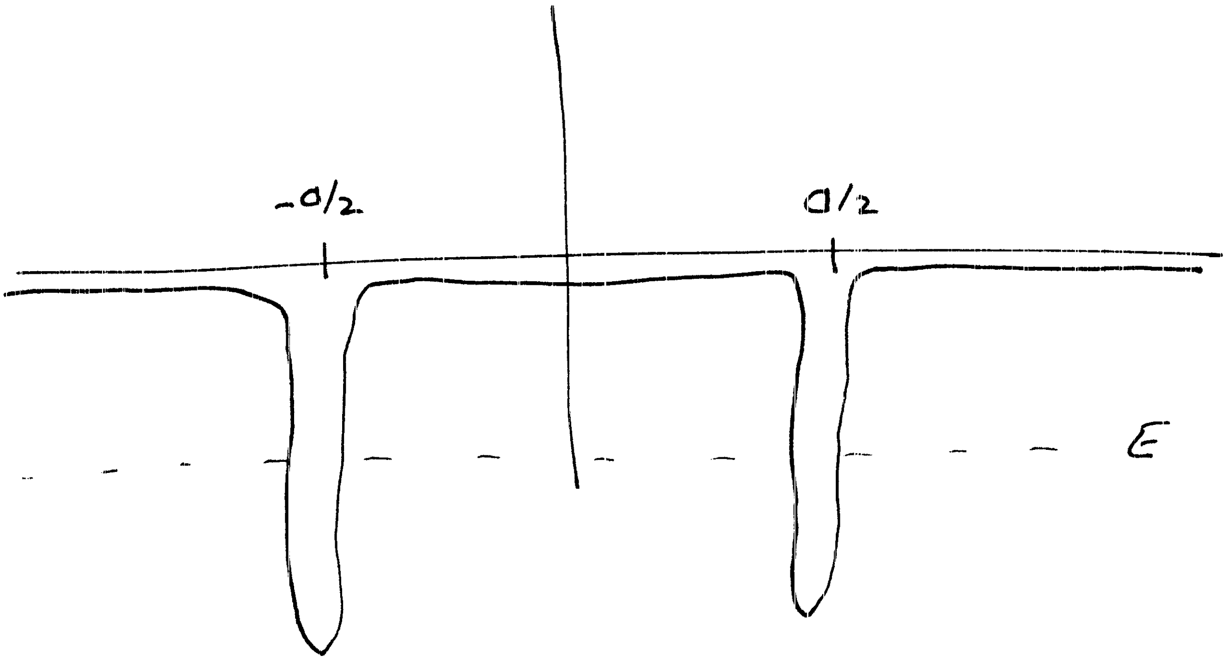
$$= 3.3\times10^{7}\,m/s$$

sort of classical

$1/10\ c$

(A2) Double delta function potential

$$V(x) = -\alpha\delta(x-a/2) - \alpha\delta(x+a/2)$$



Since $E<0$, $\phi(x) = Ae^{\pm \ell x}$, with $\ell = \sqrt{\frac{2mE}{\hbar^2}}$

everywhere.

$$\phi(x) = \begin{cases} Ae^{\ell x} & x < -a/2 \\ Be^{\ell x} + Ce^{-\ell x} & -a/2 < x < a/2 \\ De^{-\ell x} & x > a/2 \end{cases}$$

Apply BC

Continuous $x = -a/2$

$$A e^{-a\ell/2} = B e^{-a\ell/2} + C e^{a\ell/2}$$

Continuous $x = a/2$

$$B e^{\ell a/2} + C e^{-\ell a/2} = D e^{-\ell a/2}$$

Slope $x = -a/2$

$$\left.\frac{d\phi}{dx}\right|_{+} - \left.\frac{d\phi}{dx}\right|_{-} = \frac{2m}{\hbar^2}\int_{-}^{+} V(x)\phi(x)\,dx = -\frac{2m\alpha}{\hbar^2}\phi(-a/2)$$

$$\left(B\ell e^{-a\ell/2} - C\ell e^{a\ell/2}\right) - A\ell e^{-a\ell/2} = -\frac{2m\alpha}{\hbar^2} A e^{-a\ell/2}$$

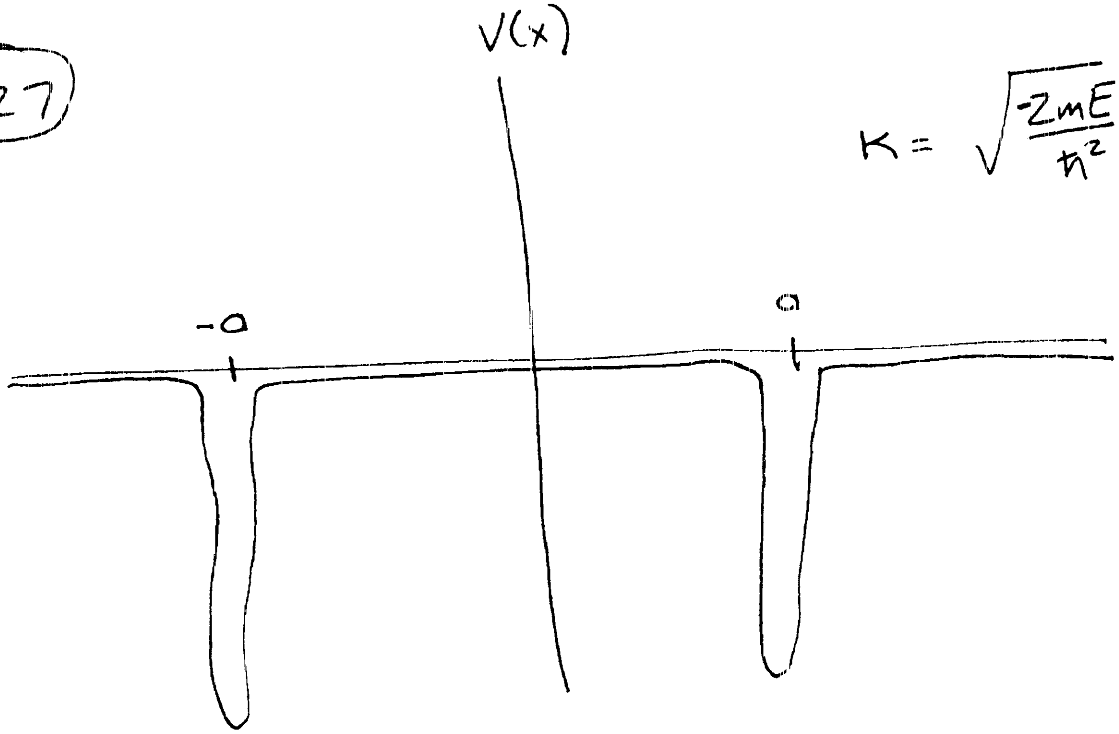
Slope $x = +a/2$

$$-D\ell e^{-\ell a/2} - \left(B\ell e^{\ell a/2} - C\ell e^{-\ell a/2}\right) = -\frac{2m\alpha}{\hbar^2} D e^{-\frac{\ell a}{2}}$$

The full solution follows but was not required. Since the well was symmetric, it exploits symmetry.

2.27

$$\kappa = \sqrt{\frac{-2mE}{\hbar^2}}$$



Even Solutions

$$\phi(x) = \begin{cases} A e^{+\kappa x} & x < -a \\ B(e^{\kappa x} + e^{-\kappa x}) & -a < x < a \\ A e^{-\kappa x} & x > a \end{cases}$$

Continuity

$x = -a$: $\quad A e^{-\kappa a} = B(e^{-\kappa a} + e^{\kappa a})$

Since function even, this also satisfied other ~~bo~~ $x = a$.

$$A = B(1 + \cancel{2}\, e^{2\kappa a})$$

Discontinuous Derivative

$$\left.\frac{d\phi}{dx}\right|_{a+} - \left.\frac{d\phi}{dx}\right|_{a-} = +\frac{2m}{\hbar^2}\int_{a^-}^{a^+} V(x)\phi(x)\,dx$$

$$= -\frac{2m\alpha}{\hbar^2}\phi(a)$$

$$-A\kappa e^{-\kappa a} - B(\kappa e^{\kappa a} - \kappa e^{-\kappa a}) = -\frac{2m\alpha}{\hbar^2} A e^{-\kappa a}$$

$$A + B(e^{2\kappa a} - 1) = \frac{2m\alpha}{\hbar^2\kappa} A$$

$$B(e^{2\kappa a} - 1) = A\left(\frac{2m\alpha}{\hbar^2\kappa} - 1\right)$$

$$= B(1 + e^{2\kappa a})\left(\frac{2m\alpha}{\hbar^2\kappa} - 1\right)$$

Cancel B

$$e^{2\kappa a} - 1 = \frac{2m\alpha}{\hbar^2\kappa} - 1 + e^{2\kappa a}\left(\frac{2m\alpha}{\hbar^2\kappa} - 1\right)$$

$$e^{2\kappa a} = \frac{2m\alpha}{\hbar^2\kappa} + e^{2\kappa a}\left(\frac{2m\alpha}{\hbar^2\kappa} - 1\right)$$

$$1 = \frac{2m\alpha}{\hbar^2\kappa} e^{-2\kappa a} + \left(\frac{2m\alpha}{\hbar^2\kappa} - 1 \right)$$

$$2 = \frac{2m\alpha}{\hbar^2\kappa} e^{-2\kappa a} + \frac{2m\alpha}{\hbar^2\kappa}$$

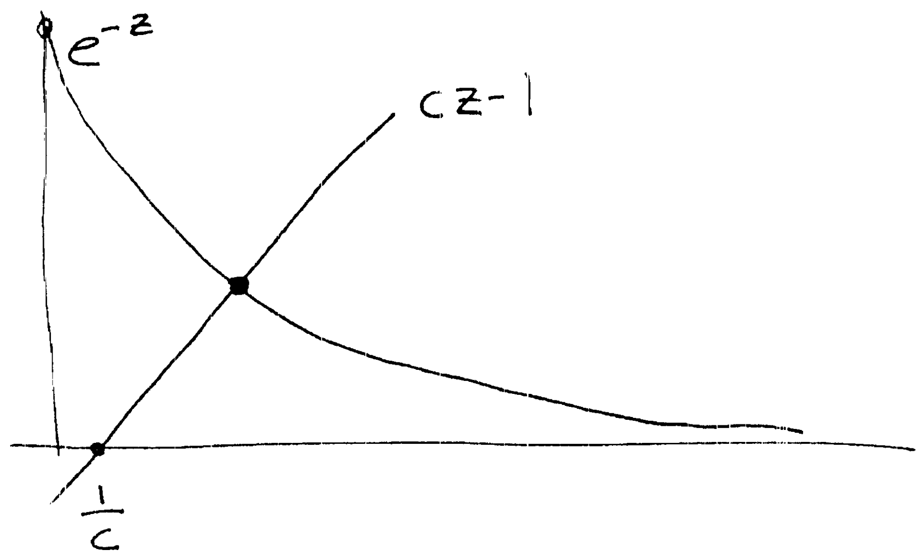
$$\boxed{\frac{\hbar^2\kappa}{m\alpha} - 1 = e^{-2\kappa a}}$$

Define $z = 2\kappa a$ $\qquad c = \dfrac{\hbar^2}{2am\alpha}$

<u>Equation becomes</u>

$$cz - 1 = e^{-z}$$

Plot



Always one intercept, only one solution always.

Use Maple's solver (fsolve, evalf)

If $\alpha = \frac{\hbar^2}{2ma} \Rightarrow c = 1 \qquad z = 1.278$

Odd solutions

$$\phi = \begin{cases} -A e^{\kappa x} & x < -a \\ B(e^{\kappa x} - e^{-\kappa x}) & -a < x < a \\ +A e^{-\kappa x} & x > a \end{cases}$$

Continuity $\quad A e^{-\kappa a} = B(e^{\kappa a} - e^{-\kappa a})$

Slope

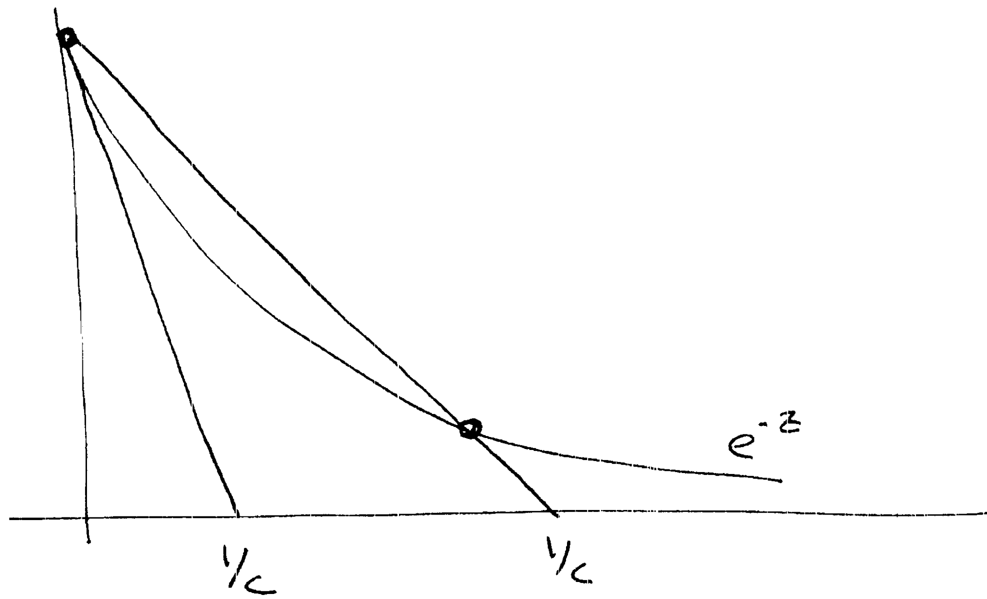
$$-\kappa A e^{-\kappa a} - B(\kappa e^{\kappa a} + \kappa e^{-\kappa a}) = -\frac{2m\alpha}{\hbar^2} A e^{-\kappa a}$$

$$\Downarrow$$

$$e^{-2\kappa a} = 1 - \frac{\hbar^2 \kappa}{m\alpha}$$

$$e^{-z} = 1 - c z$$

Plot

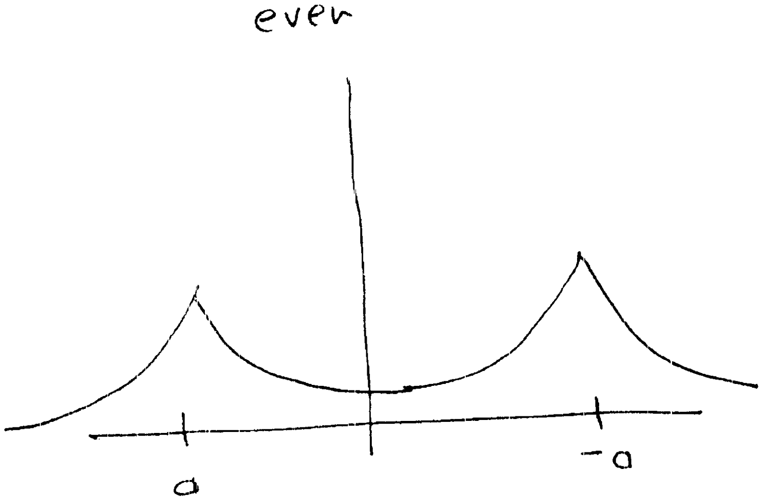


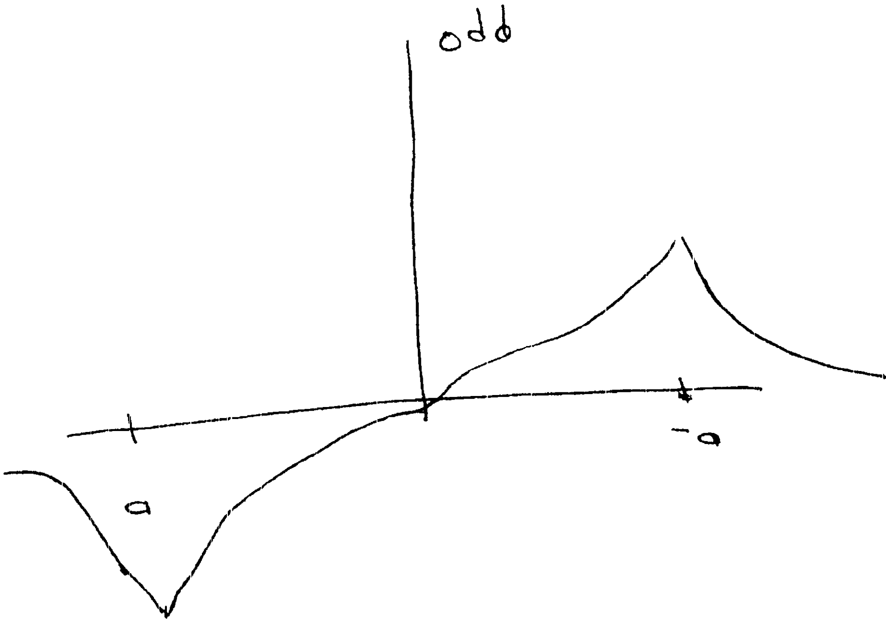
The $z=0$ intercept $\Rightarrow$ $\kappa=0$ is not normalizable.

The odd solution exists if the slope of the line is greater than the slope of e^{-z} at zero, so the odd solution exists if $-c > -1$ or if $c<1$.

The double delta function has one even solution if $c>1$ and one even and one odd solution if $c<1$.

$$c = \frac{\hbar^2}{2am\alpha}$$





If $\alpha = \frac{\hbar^2}{ma}$ $c = \frac{1}{2}$ $z = 2.2$ even

$z = 1.59$ odd

$z = 2ka$ $k = \frac{z}{2a}$

$$E = -\frac{\hbar^2 k^2}{2m} = \frac{\hbar^2}{ma^2}\left(\frac{z^2}{8}\right)$$

$$E_{even} = -0.6\left(\frac{\hbar^2}{ma^2}\right)$$

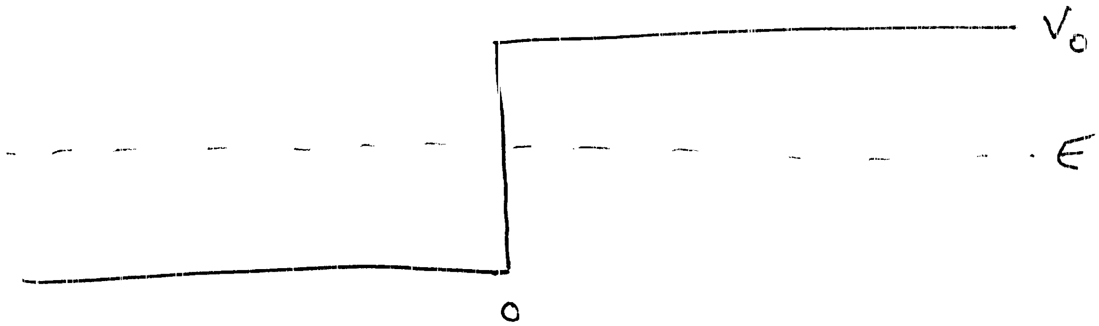
$$E_{odd} = -0.3\left(\frac{\hbar^2}{ma^2}\right)$$

If $\alpha = \dfrac{\hbar^2}{4ma} \Rightarrow c = 2$ only even solution

$$z = 0.73$$

$$E = -0.07\,\frac{\hbar^2}{ma^2}$$

(A3) Finite Step



$$\phi(x) = \begin{cases} A_I e^{ikx} + A_R e^{-ikx} & x<0 \\ A_T e^{-\ell x} & x>0 \end{cases}$$

$$k = \sqrt{\frac{2mE}{\hbar^2}} \qquad \ell = \sqrt{\frac{2m(V_0-E)}{\hbar^2}}$$

__Boundary Conditions__

__Continuity $x=0$.__

$$A_I + A_R = A_T$$

__Slope Continuous at $x=0$__

$$ikA_I - ikA_R = -\ell A_T$$

Eliminate A_T

$$ik A_I - ik A_R = -\ell (A_I + A_R)$$

$$(\ell + ik) A_I = A_R (ik - \ell)$$

$$\frac{A_R}{A_I} = \frac{\ell + ik}{ik - \ell}$$

$$R = \frac{A_R^* A_R}{A_I^* A_I} = \left(\frac{\ell + ik}{ik - \ell}\right)^* \left(\frac{\ell + ik}{ik - \ell}\right)$$

$$= \left(\frac{\ell - ik}{-ik - \ell}\right)\left(\frac{\ell + ik}{ik - \ell}\right) = 1$$

$$T = 1 - R = 0$$

(A4) $V_0 = 2.24 \text{MeV} = 3.59\times10^{-13} \text{J}$

$a = 2r = 4.28\times10^{-15} \text{m}$

Infinite Square Well

$$E = \frac{\hbar^2 k^2}{2m} = \frac{\hbar^2 \pi^2 n^2}{2ma^2}$$

For ground state, $n=1$

$$E = \frac{\hbar^2 \pi^2}{2ma^2} = \frac{(1.05\times10^{-34} \text{J}\cdot\text{s})^2 \pi^2}{2(1.67\times10^{-27} \text{kg})(4.28\times10^{-15} \text{m})^2}$$

$$= \cancel{2.34\times10^{-11}} \text{J} \quad 1.78\times10^{-12} \text{J}$$

$$= \cancel{146 \text{MeV}} = 11 \text{MeV}$$

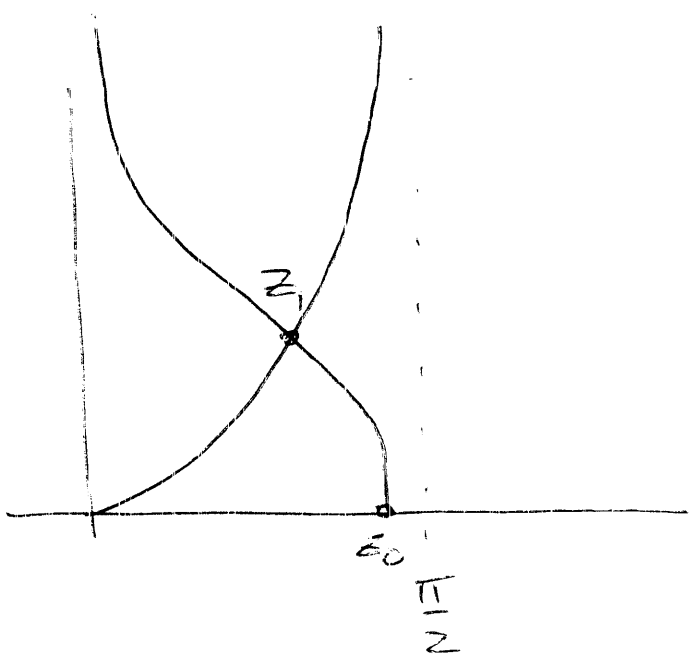
Finite Well

$V_0 = 2.24 MeV$

$= 3.59 \times 10^{-13} J$

$$z_0^2 = \frac{2ma^2V_0}{\hbar^2} = \frac{2(1.67\times10^{-27}kg)(4.28\times10^{-15}m)^2(3.59\times10^{-13}J)}{(1.05\times10^{-34}Js)^2}$$

$= 1.99$ $\qquad z_0 = 1.41 < \frac{\pi}{2}$



Find z_1

$$\left(\left(\frac{z_0}{z}\right)^2 - 1\right)^{1/2} = \tan(z)$$

Wolfram Alpha

solve sqrt((1.41/z)^2 - 1) = tan(z) for z

$$z = 0.8888$$

Maple

fsolve(sqrt((1.41/z)^2 - 1) = tan(z), z)

Energy of Ground State

$$z^2 = k^2 a^2 = \frac{2m(E+V_0)}{\hbar^2} a^2$$

$$E = \frac{\hbar^2 z^2}{2ma^2} - V_0$$

$$= 1.42\times10^{-13}\,\mathrm{J} - 3.59\times10^{-13}\,\mathrm{J}$$

$$= -2.16\times10^{-13}\,\mathrm{J}$$

$$= -1.35\,\mathrm{MeV}$$