Introduction to QM

How exactly is Quantum Mechanics (QM) different from Classical Mechanics (CM).

Consider a free particle of mass m moving through empty space.

The particle is completely described by the trajectory $\vec{r}(t)$, the particles location, as a function of time. The particles momentum, $\vec{p}$, is defined as $\vec{p} = m \frac{d\vec{r}}{dt}$

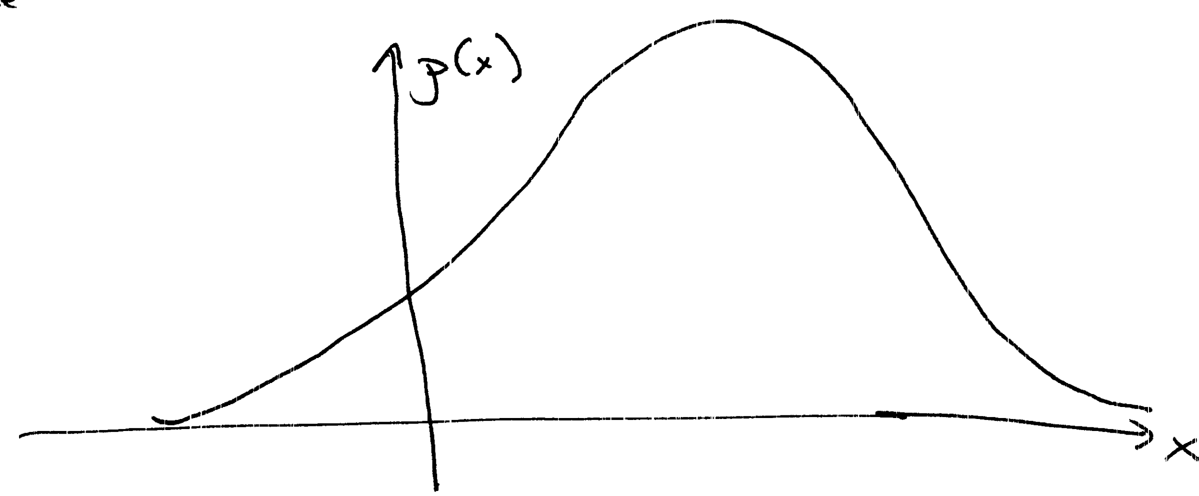
In QM, the description changes dramatically. The particles location is now only specified by a probability distribution, $\rho(\vec{r}, t)$. The particles momentum is also described by a probability distribution, $\rho(\vec{p}, t)$. The two probability distributions have a complicated relation.

Quickly recall probability

If $p(x)$ is the probability density for the random variable x the

$$\int_{-\infty}^{\infty} p(x)\,dx = 1$$

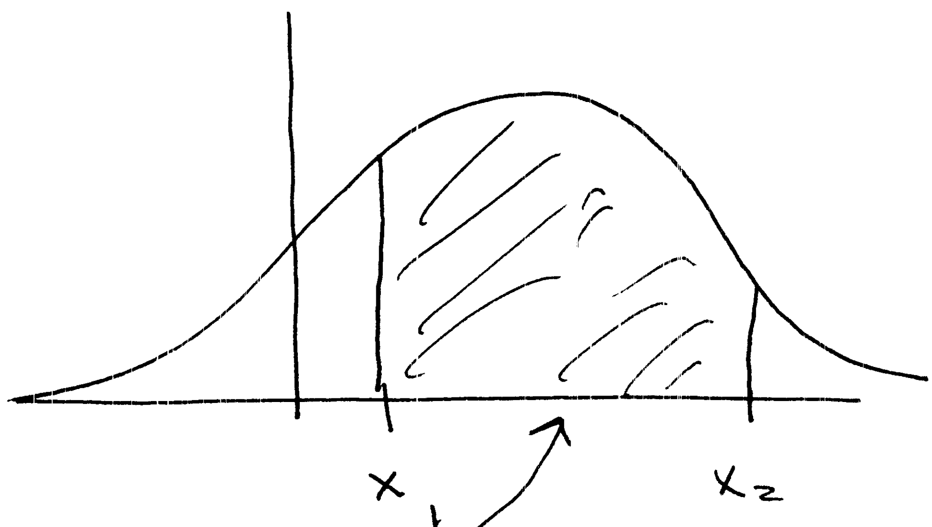
The most common $p(x)$ is the normal distribution, the Bell curve.



If the position of the particle is given by a probability function, we can no longer answer the question, "where is the particle?"

We are left answering the question, "what is the probability the particle is between x_1 and x_2?"

We answer this question by finding the area under the probability curve from x_1 to x_2.



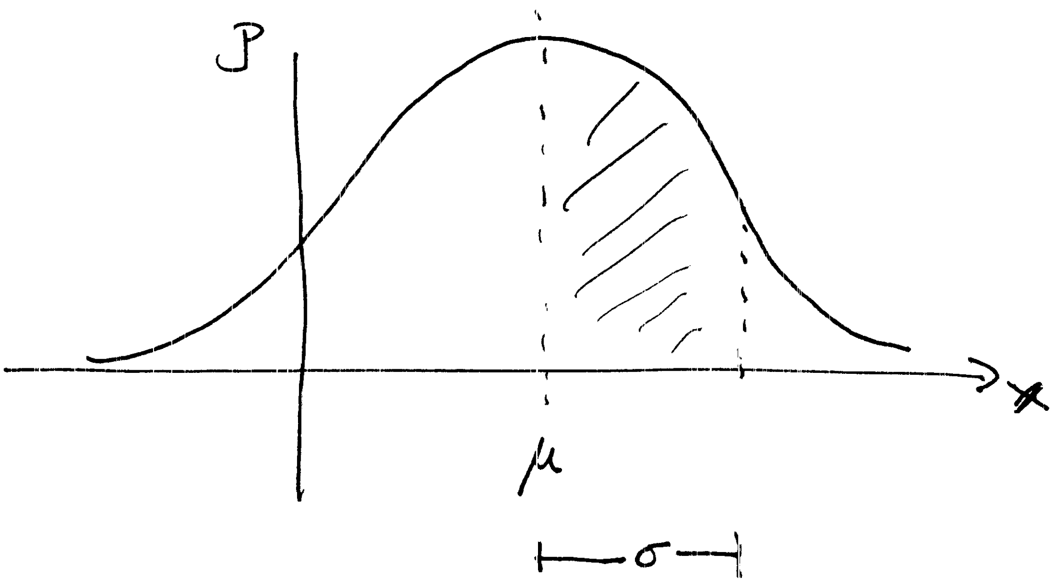
$$\mathcal{P}(x_1, x_2) = \int_{x_1}^{x_2} p(x)\,dx$$

where $\mathcal{P}(x_1, x_2)$ is the probability $x \in [x_1, x_2]$

We can characterize a probability distribution by two parameters:

μ : mean : the center of the distribution

σ : standard derviation : the width of the distribution.



$$P(\mu, \sigma) = 0.34 \implies x \in [\mu, \mu+\sigma]$$

Therefore, for a quantum particle the location
is given by $p(\vec{r}, t)$, so we can determine
on average location of the particle $\mu(t)$ and
how well we know the particle will be near
the average by $\sigma_{\vec{r}}$.

But this isn't a real difference with CM!
All measurements have uncertainty, so for a real
particle in classical mechanics we also correctly
describe the momentum and position by probability
distributions, $p(\vec{r}, t)$ and $p(\vec{p}, t)$ and we
express prediction about the location and momentum
of the particle in the form, the probability the
particle is between x_1 and x_2 is 90%.

The width of the position distribution is
a measurement of our <u>classical ignorance</u> of
the position.

QM introduces some additional and unusual features.

(1) In CM, the position and momentum can be measured independently and with careful work we can make $\sigma_{\vec{r}} \to 0$ and $\sigma_{\vec{p}} \to 0$.

In QM, uncertainty is intrinsic. Attempts to further refine the location of the particle can result in greater uncertainty in the momentum –

Uncertainty Relation

$$\sigma_x \, \sigma_p \geq \frac{\hbar}{2}$$

<u>Planck's Constant</u> $h = 6.63 \times 10^{-34}\ \mathrm{J \cdot s}$

"h-bar" $\hbar = 1.055 \times 10^{-34}\ \mathrm{J \cdot s}$

Notice, $\hbar$ is very small. For most measurements, the uncertainty relation provides no restrictions.

(2) In CM, uncertainty is somewhat independent of the physics. If we measure the position and momentum carefully, the uncertainty in each is fairly constant, for example if $\sigma_p = 0$, then σ_r is constant.

In QM, the uncertainty in location grows as a function of time as a result of the basic physics of the system.

$\Rightarrow$ As a quantum particle moves through space, its $\rho(\vec{r}, t)$ function broadens, $\sigma_r(t)$ increases with time.

(3) In CM, measurement is non-invasive; we can measure $\vec{r}$ and $\vec{p}$ to arbitrary accuracy without disturbing the system.
$\Longrightarrow$ It doesn't matter if you watch a CM system.
In QM, measurement fundamentally alters the system. If as the particle travels, we make a careful measurement of the location of particle immediately (without speed of light restrictions) changes the physical state of the system!
$\Longrightarrow$ Watching a QM system changes the system.