Postulates

Physics should, up to math difficulties, be able to answer any question posed about a physical system. Wave mechanics was kind of hodge-podge, so now we will lay out a consistent set of postulates for the theory of quantum mechanics.

All measurements in QM should be analyzable in terms of these postulates.

Before proceeding, consider the postulates of classical mechanics.

Classical Mechanics

I. The state of a mechanical system at time t is completely specified by giving mass m_i, the position $\vec{r}_i$, and the velocity $\vec{v}_i$ of each particle in the system.

II. The momentum of mass m_i is $\vec{p}_i = m_i \vec{v}_i$

III. The net force on particle m_i is defined as

$$\vec{F}_i = \frac{d\vec{p}_i}{dt}$$

IV. Forces Obey

$$\vec{F}_{12} = -\vec{F}_{21}$$

-or-

$$\frac{d\vec{p}_{12}}{dt} = -\frac{d\vec{p}_{21}}{dt}$$

$\Rightarrow$ Momentum is pairwise conserved.

V. Other fields of physics like E+M are added to CM by providing a force law.

Alternate Formulations

Lagrangian Dynamics

I. The Lagrangian of a system is defined as the difference in the kinetic and potential energy $L = T - V$

II **Hamilton's Principle** The path taken by a system is the one ~~[illegible]~~ for which the time integral of the Lagrangian is an extrema

$$\delta \int_{t_1}^{t_2} L(\vec{r}_i, \vec{v}_i, t)\, dt = 0$$

$\Rightarrow$ Equations of Motion

$$\frac{\partial L}{\partial x_i} - \frac{d}{dt}\frac{\partial L}{\partial v_i} = 0$$

III. The coordinate x_i may be any that describe the system, for example $x_i = \Theta_i$.

Hamiltons Formalism

I. Generalized Momenta

$$p_i \equiv \frac{\partial L}{\partial \dot{x}_i} = \frac{\partial L}{\partial v_i}$$

II. The <u>Hamiltonian</u> of the system is the total energy written in terms of the position and the generalized momentum

$$H(x_i, p_i, t) = T + V$$

III EOM

$$\dot{x}_i = v_i = \frac{\partial H}{\partial p_i} \qquad -\frac{dp_i}{dt} = \frac{\partial H}{\partial x_i}$$

Postulates of Quantum Mechanics

I. At time t_0, the state of the system is fully specified by a vector in a vector space $\mathbb{V}$ called the <u>state space</u>. We will denote the vector as $|\psi\rangle$.

II. Every possible measurable quantity 'a' is represented by an operator $\hat{A}$ on $\mathbb{V}$ called an <u>observable</u>.

III. The only possible results of a measurement of 'a' are the <u>eigenvalues</u> of $\hat{A}$.

<u>IV</u>. <u>If</u> $\hat{A}$ has discrete, non-degenerate eigenvalues $\{a_i\}$ then the probability of observing a_n is $|\langle a_n|\psi\rangle|^2$ where $|a_n\rangle$ is the eigenvector associated with a_n

$$\hat{A}|a_n\rangle = a_n|a_n\rangle$$

and $\langle a_n|\psi\rangle$ is the <u>inner product</u> of $|a\rangle$ and $|\psi\rangle$ in $\mathbb{V}$.

<u>V</u> (Collapse of Wave Function) If a measurement of 'a' results in a_n, then the state of the system <u>immediately</u> transforms to $|a_n\rangle$

$$|\psi\rangle \xrightarrow{a_n} |a_n\rangle$$

<u>VI</u> The state of the system evolves according to the Schrodinger Eqn

$$i\hbar \frac{d}{dt}|\psi\rangle = \hat{H}|\psi\rangle$$

where $\hat{H}$ is the observable associated with the total energy of the system, the <u>Hamiltonian</u>.

VII <u>Quantization Rules</u> The quantum mechanical operator $\hat{A}$ is formed from a classical mechanical quantity $A(\vec{r}, \vec{p}, t)$ by:

(A) Replace $\vec{r}$ with the position operator

$$\hat{\vec{R}} = (\hat{X}, \hat{Y}, \hat{Z})$$

(B) Replace $\vec{p}$ with the momentum operator

$$\hat{\vec{P}} = (\hat{P}_x, \hat{P}_y, \hat{P}_z)$$

(C) The resulting operator must be symmetrized

i.e. if $A = xp$

$$\hat{A} = \frac{\hat{X}\hat{P} + \hat{P}\hat{X}}{2}$$

VIII In the $\vec{r}$ basis, the state vector is $\langle x | \psi \rangle = \psi(x, t)$, the position operator is $\hat{\vec{R}} = (x, y, z)$, and the momentum operator $\hat{\vec{P}} = \frac{\hbar}{i} \nabla$

Generalizations of Postulate IV

I. <u>Degenerate Discrete Spectrum</u> - Suppose the value a is associate with N eigenvectors $\{|a_i\rangle\}$, then the probability to observe a is $\mathcal{P}(a) = \sum_{i=1}^{N} |\langle a_i|\psi\rangle|^2$

Note, the collapse of the wave function must also be modified to the projection of $|\psi\rangle$ onto the space spanned by $\{|a_i\rangle\}$.

II. Continuous Spectrum

If $\hat{A}$ has a continuous spectrum, then the probability density to observe a from a to $a + da$ is

$$\rho(a) = |\langle a | \psi \rangle|^2$$

$$\int_{-\infty}^{\infty} \rho(a)\, da = 1.$$

For example, the position operator has a continuous spectrum

$$\hat{X} | x \rangle = x | x \rangle$$

so $\rho(x) = |\langle x | \psi \rangle|^2 = |\psi(x,t)|^2$

or the momentum operator $\hat{P} | p \rangle = p | p \rangle$

$$\rho(p) = |\langle p | \psi \rangle|^2 = |\Psi(p,t)|^2$$