Probability

We are going to be forced to describe the behavior of particles in terms of the probability of certain events. So we need to have a bare bones understanding of probability.

Suppose we conduct an experiment that has M possible outcomes $\{a_1, a_2 \cdots a_M\}$

Suppose the experiment is performed N times and outcome a_1 is observed N_1 times.

The probability of outcome a_1 is

$$P_1 = \frac{N_1}{N}$$

where $N = N_1 + N_2 \cdots\cdots N_M$

Suppose a_i is some numeric outcome like the number rolled on a dice, the average number rolled is

$$\frac{1}{N} \sum_i a_i N_i$$

Since something happened

$$\sum P_i = 1$$

We will denote an average by $\langle \rangle$

$$\langle a \rangle = \frac{1}{N} \sum a_i N_i = \sum a_i P_i$$

If we had to bet, $\langle a \rangle$ would be the most likely outcome of the experiment, the expected outcome. We will call $\langle a \rangle$ the expectation value of the experiment.

We would also like to characterize the spread in outcomes, Δa. The distance an outcome is from the average is

$$a - \langle a \rangle$$

The average distance from the average is

$$\langle a - \langle a \rangle \rangle = \langle a \rangle - \langle a \rangle = 0$$

So we will characterize the spread by the average square distance

$$\Delta a = \sqrt{\langle (a - \langle a \rangle)^2 \rangle}$$

$$\langle (a - \langle a \rangle)^2 \rangle = \langle a^2 - 2a\langle a \rangle + \langle a \rangle^2 \rangle$$

$$= \langle a^2 \rangle - 2\langle a \rangle \langle a \rangle + \langle a \rangle^2$$

$$= \langle a^2 \rangle - \langle a \rangle^2$$

$$\Delta a = \sqrt{\langle a^2 \rangle - \langle a \rangle^2} \quad \text{- Standard deviation}$$

We will denote the standard deviation of a variable a by σ_a

$$\sigma_a = \sqrt{\langle a^2 \rangle - \langle a \rangle^2}$$

The probability P_i is the probability of a discrete outcome, but suppose our experiment measures a continuous variable, like the height of a person.

In the continuous case, we have a probability density $p(x)$ that gives the probability of finding the height in the range $x, x+dx$.

In the discrete case, the sum of the probability was $1 = \sum_i P_i$ indicating something was measured for each experiment. For a continuous probability density, this is expressed by

$$\int_{-\infty}^{\infty} p(x)\,dx = 1$$

The average of a ~~variable~~ function that depends on x, $f(x)$ is

$$\langle f(x) \rangle = \int_{-\infty}^{\infty} f(x)\,p(x)\,dx$$

So if x is the height, then the average height is

$$\langle x \rangle = \int_{-\infty}^{\infty} x\,p(x)\,dx$$

The average height squared,

$$\langle x^2 \rangle = \int_{-\infty}^{\infty} x^2 \, p(x) \, dx$$

and the standard deviation in height

$$\sigma_x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$$