PHYS 4073 - Quantum Mechanics- Test 2 - Fall 2010

All problems are worth 25 points. Turn in solutions to four of the six problems to be graded. If you turn in more than four solutions, I will grade the first four. You are allowed to drop one-half of a test, so I will take the first two problems turned in as the first half-test and the second two problems turned in as the second half-test.

1 The Hamiltonian of a two-state system in the basis $\{|1>, |2>\}$ is

$$\hat{H} = \hbar\omega \begin{pmatrix} 1 & -2i \\ 2i & 1 \end{pmatrix}$$

The system is in the state

$$|\psi> = \frac{1}{\sqrt{2}}(|1> + |2>)$$

What energies could be measured for the system in this state with what probability? What is the expectation value of the energy?

2 A system is in a simple gravitation potential $V(x) = -mgx$. Compute the uncertainty relation for the energy of the system and the position when the system is in a general state $|\psi>$. Compute the uncertainty relation for the energy of the system and the momentum when the system is in a general state $|\psi>$.

3 Show that a system moving in a simple gravitation potential $V(x) = -mgx$ has the correct classical behavior, that is show the time dependance of the average position and momentum is what you would expect classically.

4 Consider two operators of a two-state system in the basis $\{|1>, |2>\}$

$$\hat{A} = a \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$$

and

$$\hat{B} = b \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$$

Which operator or operators could represent a physical observable and why? Find the eigenvalues of both matrices. Were they what you expected? Comment. Calculate $[\hat{A}, \hat{B}]$.

5 For the space spanned by the lowest three energy states of the simple harmonic oscillator, $\{|\hbar\omega/2>, |3\hbar\omega/2>, |5\hbar\omega/2>\}$ write the matrix representing the operator

$$\hat{A} = \hat{a}_+^2 + \hat{a}_-^2$$

where $\hat{a}_+$ and $\hat{a}_-$ are the raising and lowering operators. Is $\hat{A}$ Hermitian? Justify.

6 Consider the three lowest energy states, $\{ |1>, |2>, |3> \}$ of an infinite square well with $V = \infty$ outside the range 0 to a. Write the Hamiltonian matrix and the matrix representing the position operator $\hat{X}$.