Time Evolution Operator

SE

$$i\hbar \frac{d}{dt} |\psi\rangle = \hat{H} |\psi\rangle$$

Pretend $\hat{H}$ is not an operator, then we can solve this equation by

$$|\psi(t)\rangle = \hat{U}(t, t_0) |\psi(t_0)\rangle$$

Time Evolution Operator

$$\hat{U}(t, t_0) = e^{-\frac{i\hat{H}}{\hbar}(t-t_0)}$$

$\Rightarrow$ $\hat{H}$ does depend on time

$\Rightarrow$ $\hat{U}$ is unitary, $\hat{U}^{+} = \hat{U}^{-1}$

$\Rightarrow$ $\hat{U}$ preserves the norm!

Try $\hat{U}$ on the energy eigenstates $|\phi_n\rangle$,

$$\hat{H}|\phi_n\rangle = E_n|\phi_n\rangle$$

What is $\hat{U}|\phi_n\rangle$?

$$\hat{U}|\phi_n\rangle = e^{-i\frac{\hat{H}}{\hbar}(t-t_0)}|\phi_n\rangle$$

$$= e^{-i\frac{E_n}{\hbar}(t-t_0)}|\phi_n\rangle$$

Now try a general state $|\psi\rangle$

$$\hat{U}(t,t_0)|\psi\rangle = \hat{U}\underbrace{\sum c_n|\phi_n\rangle}_{|\psi\rangle}$$

$$= \sum c_n \hat{U}|\phi_n\rangle$$

$$= \sum c_n e^{-i\frac{E_n}{\hbar}(t-t_0)}|\phi_n\rangle = |\psi(t)\rangle$$

<u>Dfn Compatable Observables</u> Observables such that $[\hat{A}, \hat{B}] = 0$. $\Rightarrow$ We can find a common set of eigenvectors $|ab\rangle$

~~Suppose~~ s.t. $\hat{A}|ab\rangle = a|ab\rangle$

and $\hat{B}|ab\rangle = b|ab\rangle$

If $\hat{A}$ and $\hat{H}$ are compatable, let $|\phi_n'\rangle = |na\rangle$ be the compatable eigenvectors, then

$$|\psi(t)\rangle = \sum c_n' e^{-i\frac{E_n}{\hbar}(t-t_0)} |na\rangle$$

Suppose $\hat{B}$ and $\hat{H}$ are not compatable, $[\hat{H}, \hat{B}] \neq 0$

Let $\{|b_i\rangle\}$ be the orthonormal eigenvectors of $\hat{B}$.

<u>Closure</u> $\hat{1} = \sum_i |b_i\rangle\langle b_i|$

$$|\psi(t)\rangle = \sum_n c_n e^{-i\frac{E_n}{\hbar}(t-t_0)} |\phi_n\rangle$$

insert $\hat{1}$

$$|\psi(t)\rangle = \sum_{nj} c_n e^{-i\frac{E_n}{\hbar}(t-t_0)} |b_j\rangle\langle b_j|\phi_n\rangle$$

$$\equiv \sum_j b_j(t)|b_j\rangle$$ which we can do because $\{|b_j\rangle\}$ span.

<u>by observation</u>

$$b_j(t) = \sum_n c_n e^{-i\frac{E_n}{\hbar}(t-t_0)} \langle b_j|\phi_n\rangle$$

$\Rightarrow$ $b_j(t)$ no longer constant

$\Rightarrow$ $b_j(t)$ depends on a mix of frequencies