Vectors

$|\psi\rangle$ is the state vector which describes a quantum system. It lives in a vector space. As $|\psi\rangle$ evolves, it changes into different vectors in the space but never leaves the space.

The model for a vector space is the normal vectors in 3 dimensional space, like electric field vectors $\vec{E}$.

Our vectors will be written as $|a\rangle$, and are called kets. The "a" is a label for the ket.

Dfn Vector Space A vector space is composed of a collection of vectors $\{|a\rangle, |b\rangle, |c\rangle \ldots\}$ and scalars (numbers) $\{c_1, c_2 \ldots\}$ s.t.

1. There is an operation vector addition s.t. the sum of any two vectors is another vector in the space

$$|a\rangle + |b\rangle = |c\rangle$$

For 3 space, $\vec{E}_1 + \vec{E}_2 = \vec{E}$

2. There is an operation, scalar multiplication, s.t. that the product of a scalar and a vector is a vector

$$|c\rangle = c_1 |a\rangle$$

in 3space $4 \cdot \vec{E}_1 = \vec{E}_2$

3. Vector addition is commutative and associative.

$$|a\rangle + |b\rangle = |b\rangle + |a\rangle$$

$$(|a\rangle + |b\rangle) + |c\rangle = |a\rangle + (|b\rangle + |c\rangle)$$

4. Scalar multiplication is distributive

$$c_1(|a\rangle + |b\rangle) = c_1|a\rangle + c_2|b\rangle$$

5. There exists a null vector $|0\rangle$ s.t.

$$|a\rangle + |0\rangle = |a\rangle$$

6. Each vector $|a\rangle$ has an inverse also in the space

$$|a\rangle + |-a\rangle = |0\rangle$$

Note, $-a$ is a label so this could have been written

$$|a\rangle + |bob\rangle = |0\rangle$$

7 Scalar multiplication is associative

$$c_1(c_2|a\rangle) = (c_1c_2)|a\rangle$$

8 Multiplication by 0 and 1 behave correctly

$$0|a\rangle = |0\rangle$$

$$1|a\rangle = |a\rangle$$

You should check that normal vectors

$$\vec{E} = (a, b, c)$$

obey these conditions.

Dfn Linear Combination - Vector addition with scalar multiplication.

$$c_1|a\rangle + c_2|b\rangle + c_3|c\rangle$$

Dfn Linear Independence The vector $|c\rangle$ is linearly independent from the set $\{|a_1\rangle, |a_2\rangle \cdots\}$ if there is no linear combination s.t.

$$|c\rangle = c_1|a_1\rangle + c_2|a_2\rangle + \cdots$$

Dfn Spanning the Space - A set of vectors $\{|a_1\rangle, |a_2\rangle \cdots\}$ spans the vector space if every vector in the space is a linear combination of the spanning vectors

$$|a\rangle = c_1|a_1\rangle + c_2|a_2\rangle + \cdots$$

for all $|a\rangle \in \mathbb{V}$.

Basis for Vector Space - Any linearly independent set of vectors that spans the space.

Dimension of the Space - The number of basis vectors.

Ex The set $\hat{x}, \hat{y}, \hat{z}$ spans 3 space.
All vectors are a linear combination of $\hat{x}, \hat{y}, \hat{z}$ $\Rightarrow$ $\vec{V} = c_1 \hat{x} + c_2 \hat{y} + c_3 \hat{z}$.
There are 3 basis vectors, so the dimension of normal space is 3.

Components of a Vector - If the basis for $\mathbb{V}$ is ordered, $\{ |a_1\rangle, |a_2\rangle, \dots \}$ then the N-tuple $(c_1, c_2 \dots c_n)$ s.t.

$$|a\rangle = c_1 |a_1\rangle + c_2 |a_2\rangle + \dots + c_n |a_n\rangle$$

uniquely represents $|a\rangle$. The c_i are called the components of $|a\rangle$ along $|a_i\rangle$.