Knowledge Model

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1 Mathematics

Principle (K1) : **coordinate system** A coordinate system establishes the positive direction along a set of axes.
Corollary (K2) : **position along axis** In one dimension, the position of an object can be specified by indicating a numeric location along an axis.

Principle (K3) : **vector** A vector has both magnitude and direction
Corollary (K4) : **magnitude and direction** A vector may be specified by giving a positive magnitude and some indication of direction.

Corollary (K5) : **direction positive-negative** In one dimension, the direction of a vector can be indicated by a positive or negative in reference to a coordinate system.

Corollary (K6) : **direction north-south** In one dimension, the direction can be specified using points on a compass.

Corollary (K76) : **direction left-right** In one dimension, the direction can be specified using points on a compass.

Corollary (K7) : **magnitude** The magnitude of a vector is its length and is a positive number.

Corollary (K8) : **vector as signed number** In one dimension, the a vector can be indicated as a signed number in reference to a coordinate system. In this case, it is one component of the three dimensional vector.

Principle (K78) : **two-dimensional graph** A curve presented graphically in two dimensions.
Corollary (K79) : **horizontal line is zero slope** A horizontal line represents constant slope.

Corollary (K80) : **recognize positive slope** An upward directed curve has positive slope.

Corollary (K81) : **recognize negative slope** A downward directed curve has positive slope.

Corollary (K82) : **recognize positive curvature** A concave up curve has positive curvature and a positive second derivative.

Corollary (K83) : **recognize negative curvature** A concave down curve has negative curvature and a negative second derivative.

Corollary (K84) : **recognize zero curvature** A flat segment has a zero second derivative.

2 Kinematics

Term : **Object** - Source: GS An object is considered a rigid body with fixed mass.

Term : **Kinematics** - Source: SV8
Direct Quote: The part of dynamics that describes motion without regard to its causes is called kinematics.

Term : **1D kinematics** - Source: SV8 The application of kinematics to objects that move in only one dimension.

Term : **trajectory** - Source: The path an object traverses at it moves from point A to point B.

2.1 Position and Time

Term : **elapsed time** - Source: F1 The time difference between two events such as the beginning and ending of a trajectory. Synonym: duration

Principle (K9) : **position** The position of an object is the location in space of the object. If a coordinate system is specified, the position is the coordinates of the object. The position may be be written as a function of time, $\vec{r}(t)$. Alternate Term : - Source: instantaneous position
Corollary (K10) : **position 1D** In one dimension if a coordinate system is specified, the position of an object is the one-dimensional coordinates of the object. The position can be written as a function of time as $x(t)$.

Principle (K78) : **time** The position of an object can be parameterized by time.

2.2 Displacement and Distance

Principle (K11) : **displacement** An object's displacement is the change in its position vector, or

$$\Delta\vec{r} = \vec{r}_f - \vec{r}_i = \vec{r}_{if}$$

The use of vector in the above is redundant - position is a vector.
Corollary (K12) : **displacement 1D** In one dimension, if a coordinate system is specified, the displacement is $\Delta x = x_f - x_i = x_{if}$.

Principle (K13) : **distance** The distance between two points is the magnitude of the displacement between the points.
Corollary (K14) : **distance 1D** In one dimension, the distance between two points is the absolute value of the difference in the position of the points.

Corollary (K15) : **Pythagorean theorem** The distance between two points can found by applying Pythagorean theorem.

Corollary (K16) : **distance is positive** Because distance is a magnitude, it must be positive.

Principle (K17) : **total distance traveled** The total distance traveled is the length of the trajectory.

Model : **trajectory diagram** - Source: K18 This type of diagram draws the physical trajectory as a directed curve.
Corollary (K74) : **Relation trajectory curve and total distance traveled** The length of the trajectory curve is the total distance traveled. Model Supports Knowledge Element: (K17) total distance traveled

Corollary (K19) : **Relation trajectory curve and displacement** The vector from the beginning point of a trajectory to the end point is the displacement during the trajectory. Model Supports Knowledge Element: (K12) displacement 1D

2.3 Velocity and Speed

Principle (K20) : **velocity** The velocity at time t is the derivative of the displacement

$$\vec{v} = \frac{d\vec{r}}{dt}$$

Corollary (K21) : **velocity 1D** The velocity at time t is the derivative of the displacement

$$v = \frac{dx(t)}{dt}$$

Alternate Term : **instantaneous velocity** - Source:
Corollary (K22) : **velocity at turning point is zero.** At the point an object reverses direction, the velocity is zero.

Corollary (K23) : **the integral of the velocity is the displacement.** The integral of the velocity from time A to time B is the displacement of the object between those times. Model Supports Knowledge Element: (K12) displacement 1D

Corollary (K24) : **the position does not determine the sign of the velocity.** It is impossible knowing only the position at one point to determine the sign of the velocity.

Principle (K25) : **average velocity** The average velocity, $\vec{v}_{ave}$, between time t_1 and t_2 is the displacement from t_1 to t_2, $\vec{r}_{12}$ divided by the change in time $\Delta t = t_2 - t_1$

$$\vec{v}_{ave} = \frac{\vec{r}_{12}}{\Delta t}$$

Corollary (K26) : **1D average velocity** In one dimension, the average velocity is v_{ave}, between time t_1 and t_2 is the displacement from t_1 to t_2, x_{12} divided by the change in time $\Delta t = t_2 - t_1$

$$v_{ave} = \frac{x_{12}}{\Delta t}$$

Principle (K27) : **speed** The speed of an object at an instant in time is the magnitude of its velocity. As a magnitude, the speed must be positive. The speed is a scalar. Alternate Term : **instantaneous speed** - Source:

Principle (K28) : **average speed** The average speed of an object is the total distance traveled divided by the total time required to travel that distance.
Corollary (K29) : **average speed vs. average velocity** The average speed and the magnitude of the average velocity are not necessarily the same. Model Supports Knowledge Element: (K26) 1D average velocity

Model : **motion diagram** - Source: K30 This type of diagram gives the location of an object at a set of points usually equally spaced in time. Some other feature of the motion is sometimes indicated such as the velocity or acceleration. The diagram can be one or more dimensional.
Corollary (K31) : **1D average velocity and motion diagrams** In one dimension, the average velocity is inversely proportional to the distance between points in a motion graph. Model Supports Knowledge Element: (K26) 1D average velocity Model Supports Knowledge Element: (K21) 1D velocity

Corollary (K32) : **equally spaced points represents constant velocity** If all points in a motion diagram are the same distance apart, the object is moving with constant velocity. Model Supports Knowledge Element: (K26) 1D average velocity Model Supports Knowledge Element: (K21) 1D velocity

Model : **position-time graph** - Source: K33 This presents a two-dimensional plot of position versus time.
Corollary (K34) : **slope is velocity** The slope at each point on the position-time graph is the instantaneous velocity at that time. Model Supports Knowledge Element: (K21) 1D velocity Model Supports Knowledge Element: (K10) position 1D

Corollary (K35) : **straight line represents constant velocity** If the position-time curve is a straight line, then the velocity is constant. Model Supports Knowledge Element: (K21) 1D velocity Model Supports Knowledge Element: (K10) position 1D

Corollary (K36) : **Turning points position-time** The point where the position-time graph is maximum or minimum is the turning point. Model Supports Knowledge Element: (K22) velocity at turning point is zero Model Supports Knowledge Element: (K10) position 1D

Corollary (K37) : **Crossing points position-time where objects same location** The point where two position-time graph cross represents points where the object are at same location. Model Supports Knowledge Element: (K10) 1D position

Corollary (K38) : **Velocity at turning point -** $x-t$ The velocity at a turning point is zero; the graph has zero slope. Model Supports Knowledge Element: (K22) velocity at turning point is zero Model Supports Knowledge Element: (K10) position 1D

Corollary (K39) : **Curvature position-time graph is acceleration** The acceleration is the second derivative of the position vs. time. Model Supports Knowledge Element: (K54) relation of acceleration and displacement Model Supports Knowledge Element: (K10) position 1D

Corollary (K40) : **acceleration and concavity** For a position vs. time graph, a concave up segment has negative acceleration; concave down positive acceleration. Model Supports Knowledge Element: (K54) relation of acceleration and displacement Model Supports Knowledge Element: (K10) position 1D

Corollary (K41) : **1D average velocity and slope** In one dimension, the average velocity is the slope of the line connecting the two time points in the position vs. time graph. Model Supports Knowledge Element: (K26) 1D average velocity Model Supports Knowledge Element: (K10) position 1D

Model : **velocity-time graph** - Source: K42 This presents a plot of velocity versus time for motion in one-dimension.
Corollary (K43) : **slope of velocity vs. time graph is the acceleration** The slope at each point on the velocity-time graph is the instantaneous acceleration at that time. Model Supports Knowledge Element: (K52) 1D acceleration Model Supports Knowledge Element: (K21) 1D velocity

Corollary (K44) : **straight line represents constant acceleration** If the velocity-time curve is a straight line, then the acceleration is constant. Model Supports Knowledge Element: (K52) 1D acceleration Model Supports Knowledge Element: (K21) 1D velocity

Corollary (K45) : **horizontal line represents constant velocity** If the velocity-time curve is a straight horizontal line, then the acceleration is zero. Model Supports Knowledge Element: (K52) 1D acceleration Model Supports Knowledge Element: (K21) 1D velocity

Corollary (K46) : **area under velocity-time graph is displacement** The area under the velocity-time graph is the displacement. Model Supports Knowledge Element: (K23) the integral of the velocity is the displacement Model Supports Knowledge Element: (K21) 1D velocity

Corollary (K47) : **Turning points velocity-time** The point where the velocity-time graph crosses zero is the turning point. Model Supports Knowledge Element: (K22) velocity at turning point is zero. Model Supports Knowledge Element: (K21) 1D velocity

Corollary (K48) : **Average acceleration and velocity-time graphs** The slope of the line between two point on a velocity-time graph is the average acceleration. Model Supports Knowledge Element: (K60) average acceleration 1D Model Supports Knowledge Element: (K21) 1D velocity

2.4 Constant Velocity Kinematics

Term : **constant velocity** - Source: F1 The magnitude and direction of the velocity do not change with time.

Principle (K49) : **constant velocity kinematic equation** If the velocity is constant, then the position as a function of time is given by

$$\vec{r}(t) = \vec{r}_0 + \vec{v}t$$

where $\vec{r}_0$ is the position of the object at $t = 0$ and $\vec{v}$ is the constant velocity.
Corollary (K50) : **constant velocity kinematic equation 1D** For motion in one dimension along the x-axis, if the velocity is constant and one dimensional, then the position as a function of time is given by

$$x(t) = x_0 + vt$$

where x_0 is the position of the object at $t = 0$ and v is the constant velocity in the x direction.

2.5 Acceleration

Term : **deceleration** - Source: YG8 When an objects speed is decreasing, the object is decelerating. YG8 excludes this term as too ambiguous.

Principle (K51) : **acceleration** The acceleration at time t is the derivative of the velocity

$$\vec{a} = \frac{d\vec{v}}{dt}$$

Corollary (K52) : **acceleration 1D** The acceleration at time t is the derivative of the velocity

$$a = \frac{dv}{dt}$$

Alternate Term : **instantaneous acceleration** - Source: YF13
Corollary (K53) : **relation of signs of acceleration and velocity** If the acceleration and the velocity have the same sign, then the magnitude of the velocity increases with time. If the acceleration and velocity have opposite signs, then the magnitude of the velocity decreases until it reaches zero and then increases. Note, this is true if the vectors are co-linear and true component by component if not.

Corollary (K77) : **qualitative trajectory from signs of acceleration and velocity** Predict the trajectory or a particle if the acceleration and velocity are parallel or opposite. Model Supports Knowledge Element: (K53) relation of signs of acceleration and velocity

Corollary (K54) : **relation of acceleration and displacement** The acceleration is the second derivative of the displacement with respect to time.

Corollary (K55) : **acceleration at turning point is non-zero.** At the point an object reverses direction, the acceleration is non-zero.

Corollary (K56) : **direction of velocity and change in velocity cannot be determined from acceleration** The sign of the acceleration cannot be used to determine the sign of the velocity on its own or whether the object is speeding up or slowing down.

Corollary (K57) : **the magnitude or direction of acceleration cannot be determined from velocity at point** The sign of the velocity cannot be used to determine the sign of the acceleration on its own.

Corollary (K58) : **integral of acceleration is the change in velocity** The integral of the acceleration with respect to time is the change in velocity.

Naive Reasoning: If velocity zero then acceleration is zero.

Principle (K59) : **average acceleration** The average acceleration between time t_1 and t_2 is

$$\vec{a}_{ave} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$$

where $\vec{v}_1$ is the velocity at time t_1 and $\vec{v}_2$ is the velocity at t_2.
Corollary (K60) : **average acceleration 1D** In one dimension, the average acceleration between time t_1 and t_2 is

$$a_{ave} = \frac{v_2 - v_1}{t_2 - t_1}$$

where v_1 is the velocity at time t_1 and v_2 is the velocity at t_2.

Model : **acceleration-time graph** - Source: K61 This presents a plot of acceleration versus time for motion in one-dimension.
Corollary (K62) : **slope and changing acceleration** The slope of the acceleration-time graph is the rate of change of acceleration. Model Supports Knowledge Element: (K52) 1D acceleration

Corollary (K63) : **horizontal line represents constant acceleration** If the acceleration-time curve is a horizontal line, then the acceleration is constant. Model Supports Knowledge Element: (K52) 1D acceleration

Corollary (K64) : **area under acceleration time curve is the change in velocity.** The area under acceleration time curve is the change in velocity. Model Supports Knowledge Element: (K58) integral acceleration is the change in velocity

3 Constant Acceleration Kinematics

Term : **constant acceleration** - Source: SV8 The magnitude and direction of the acceleration does not change with time.

Principle (K66) : **constant acceleration velocity equation** If the acceleration is constant, then the velocity as a function of time is given by

$$\vec{v}(t) = \vec{v}_0 + \vec{a}t$$

where $\vec{v}_0$ is the velocity of the object at $t = 0$ and $\vec{a}$ is the constant acceleration.

Corollary (K65) : **constant and instantaneous acceleration** For an object moving with constant acceleration, the instantaneous acceleration equals the average acceleration at all points.

Corollary (K67) : **constant acceleration velocity equation 1D** If the acceleration is constant and the motion is in one dimension, then the velocity as a function of time is given by

$$v(t) = v_0 + at$$

where v_0 is the velocity of the object at $t = 0$ and a is the constant acceleration.

Corollary (K68) : **average velocity under constant acceleration** If the acceleration is constant, then the average velocity over a given time interval is the average of the initial and final velocities

$$\vec{v}_{ave} = \frac{\vec{v}_1 + \vec{v}_2}{2}$$

where v_1 is the velocity at t_1 and $\vec{v}_2$ is the velocity at t_2.

Principle (K75) : **constant acceleration displacement equation**
Corollary (K69) : **displacement under constant acceleration (velocity form)** If the acceleration is constant, then the displacement $\vec{r}_{12}$ from time t_1 to time t_2 is

$$\vec{r}_{12} = \vec{v}_{ave}(t_2 - t_1)$$

Corollary (K70) : **displacement under constant acceleration (acceleration form)** If the acceleration is constant, then the displacement $\vec{r}_{12}$ from time t_1 to time t_2 is

$$\vec{r}_{12} = \vec{v}_1(t_2 - t_1) + \frac{1}{2}\vec{a}(t_2 - t_1)^2$$

Corollary (K71) : **1D displacement under constant acceleration (velocity form)** If the acceleration is constant, then the displacement in one dimension $\Delta x = x_{12}$ from time t_1 to time t_2 is

$$\Delta x = v_{ave}(t_2 - t_1)$$

Corollary (K72) : **1D displacement under constant acceleration (acceleration form)** If the acceleration is constant, then the 1D displacement Δx from time t_1 to time t_2 is

$$\Delta x = v_1(t_2 - t_1) + \frac{1}{2}a(t_2 - t_1)^2$$

Corollary (K73) : **1D displacement, velocity, acceleration relation** If the acceleration is constant, then

$$v_2^2 = v_1^2 + 2a\Delta x$$

4 Projectile Motion

Term : **free fall** - Source: G1 An object is in free fall if the only force acting on it is gravity.

Term : **projectile motion** - Source: G1 Motion in one or more dimension when the only force on an object is gravity.

Principle (K79) : **acceleration of gravity** Near the earth's surface the acceleration of an object in free fall is approximately constant; the acceleration of gravity is $g = 9.81\text{m/s}^2$

5 Relative Motion

Term : **reference frame** - Source: G1 A reference frame is defined by an origin and a set of coordinate axes.

Term : **fixed reference frame** - Source: G1 A reference frame where origin is not moving. Synonym: stationary reference frame

Principle (K80) : **relative velocity** The velocity of an object relative to another object can be wriiten in terms of both object motion in reference to a fixed reference frame. Let $\vec{v}_1^f$ be the velocity of object 1 with respect to a fixed reference frame and $\vec{v}_2^f$ be the velocity of object 2 with respect to the same fixed reference frame. The velocity of object 2 relative to object 2 is $\vec{v}_{12}^{rel} = \vec{v}_2^f - \vec{v}_1^f$