# Quantum Mechanics Fall 2003- Homework Set 2

**Due Weds Sept 17, 2003 at 5:00pm**

## Additional Problems

**Problem A1** Consider a particle of mass $m$ moving in a one dimensional potential $V = \frac{1}{2}kx^2$.

**(a)** Write the Classical Hamiltonian of the system, then the quantum mechanical Hamiltonian.

**(b)** Calculate the time rate of change of the average position $< X >$ and the average momentum $< P >$.

**(c)** Write the uncertainty relation for the momentum and energy.

**(d)** Solve the two equations in (b) for the average trajectory of the particle.

**Problem A2** Consider a 3 state system spanned by the vectors $\{|1>, |2>, |3>\}$. The hamiltonian of the system (in this basis is),

$$\hat{H} = \hbar\omega \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{pmatrix}$$

The system is in state $|\psi(0)> = \frac{1}{2}|1> + \frac{1}{\sqrt{2}}|2> + \frac{1}{2}|3>$ at time $t = 0$.

**(a)** Is the Hamiltonian Hermitian?

**(b)** If the energy of the system is measured what are the possible outcomes for this state vector and with probabilities?

**(c)** Calculate $|\psi(t)>$.

Now consider the observables.

$$\hat{A} = a \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

$$\hat{B} = b \begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

**(d)** Which of the following sets of operators are complete sets of commuting observables? $\{H\}$, $\{A\}$, $\{A, B\}$, $\{A, H\}$, or $\{H, B\}$. Justify.

**(e)** Calculate the expectation value of $B$ as a function of time.

**(f)** For $|\psi(0)>$, what outcomes of a measurement of $B$ are possible and with what probabilities?

**Problem A3** Consider the system in the previous problem.

**(a)** Calculate an uncertainty relation for $\hat{H}$ and $\hat{B}$.

**(b)** Using Ehrenfest's Thm (generalized) calculate the time rate of change of $< \hat{B} >$.

## Griffith Problems

- 3.4
- 3.9
- 3.10
- 3.22
- 3.23
- 3.39
- 3.58

## Cohen-Tannoudji Problems - in $H_{II}$

Problem CT 2.3 The state space of a certain physical system is three dimensional with orthonormal basis vectors $|u_1>$, $|u_2>$, and $|u_3>$. Let the kets $|\psi_0>$ and $|\psi_1>$ be defined as:

$$|\psi_0> = \frac{1}{\sqrt{2}}|u_1> + \frac{i}{2}|u_2> + \frac{1}{2}|u_3>$$

$$|\psi_1> = \frac{1}{\sqrt{3}}|u_1> + \frac{1}{\sqrt{3}}|u_3>$$

**(a)** Are the kets normalized, if not normalize the kets.

**(b)** Write the operator representing the projector onto $|\psi_0>$.

**(c)** Project $|\psi_1>$ onto $|\psi_0>$.

**(d)** Calculate the matrix representing the projector onto $|\psi_0>$

**(e)** Use the matrix to show you get the same result as in (c).